VORTICAL STRUCTURES AND WALL TURBULENCE
Paolo Orlandi: A vortical and turbulent life

SCALING OF THE SHEAR LAYERS CONSTITUTING THE VORTICITY COMPONENTS IN A TURBULENT CHANNEL FLOW

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Happy Birthday to someone who is forever young.

- Beginning of 2000’s visiting LEGI-Grenoble (Skiing☺)
- Doing experiments of active control through localized unsteady blowing ➔ Frustrated to not experimentally detect the vortex induced by…
- Paolo set-up his code in our Work Stations➔ Could detect the structure after a while through DNS
- ➔ DOING DNS since..

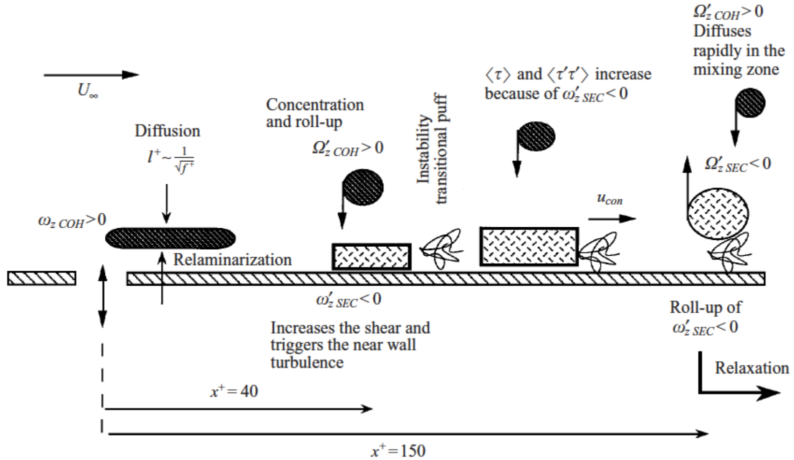


FIGURE 22. Mechanism at high imposed blowing frequencies.

J. Fluid Mech. (2001), *vol.* 439, *pp.* 217–253.

DNS

*Large computational domain as in (Hoyas, Jiménez 2006)

** NS with Dispersion Relation Preserving spatial schemes

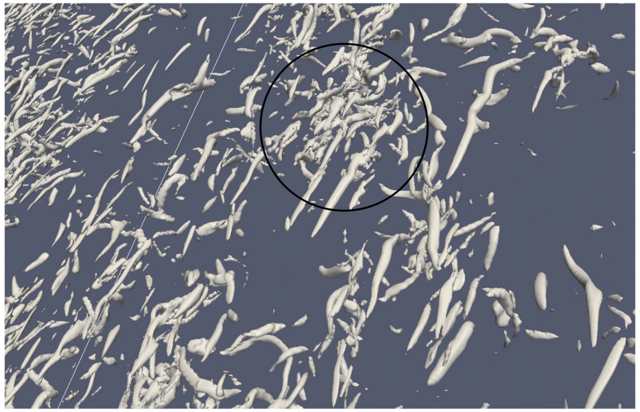
Re_τ	Re_τ actual	Resolution (N_xxN_yxNz)	Δx^+	Δy^+	Δz^+	L_x/h	L_z/h	CFL
180	177.73	771x129x387	8.80	0.49 (0.31η) 5.59 (1.52 η)	5.84	12π	4 π	0.24
395	388.77	1691x283x849	8.81	0.48 (0.33η) 5.57 (1.26η)	5.85	12π	4 π	0.26
590	580.01	1651x423x1113	8.98	0.48 (0.34η) 5.56 (1.15η)	5.00	8 π	3 π	0.33
1100	1090.82	3079x789x2075	8.98	0.48(0.34η) 5.55 (0.98η)	5.00	8 π	3 π	0.35

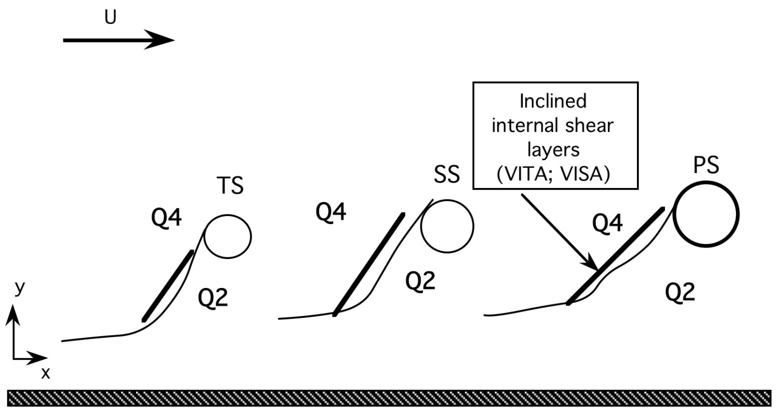
Introduction

LSM-VLSM: MAJOR TURBULENT STATISTICS ARE RE DEPENDENT

- ***Clustering*** of quasi-streamwise vortices➔ LSM

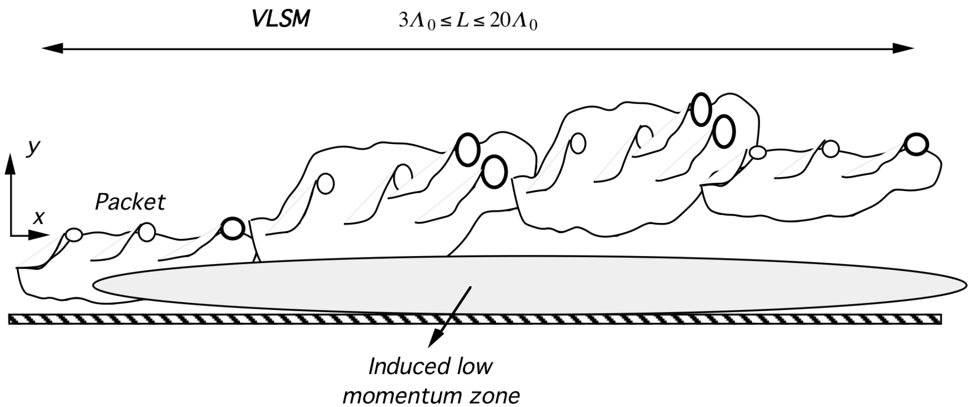
(takes place at all Reynolds numbers: Tardu (1994, 95, 2002; Adrian's group, 1999…)





DNS Reto=600

Amalgamation of packets➔ VLSM (Adrian's group, Jiménez group, Marusic's group)



TRANSPORT 50 % of shear stress.

Mainly at the median point of log-layer

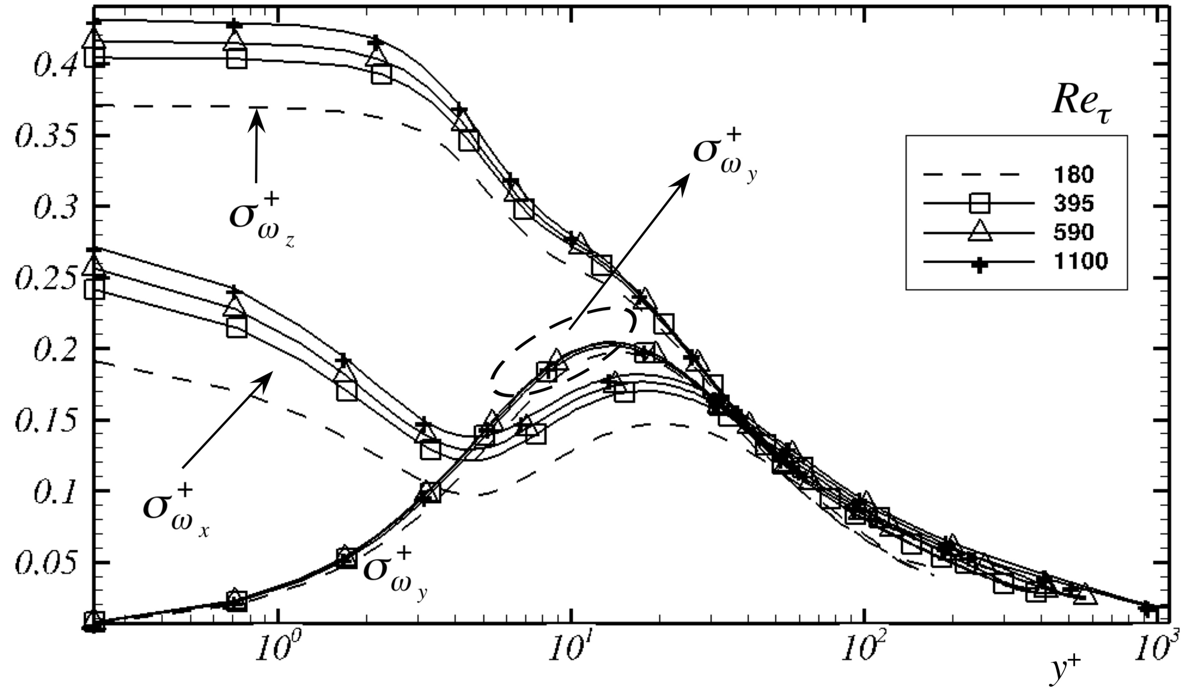
$$y^+_{R=0} \equiv y^+_M = 3.9 Re_\tau^{1/2}$$

TRANSPORT IS NOT (?) CONTRIBUTION !!

EXCEPT THE WALL NORMAL VORTICITY INTENSITY

Turbulent intensities of the vorticity components in inner variables

$$\text{Streamwise vorticity}: \omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z};\ \text{Spanwise vorticity}: \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y};\ \text{Wall normal vorticity}: \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$$



WHY the WALL NORMAL VORTICITY IS INSENSITIVE TO Re?

Asymptotic behavior of fluctuating velocity field *u* and *w* near the wall

- Constant shear stress zone (Townsend, Perry & Marusic…):

$$\frac{\overline{uu}}{\overline{u}_\tau^2}\left(y^+\right) = -A_{uu}\ln\frac{y}{h} + B_{uu}(\Pi)$$

$$\frac{\overline{ww}}{\overline{u}_\tau^2}\left(y^+\right) = -A_{ww}\ln\frac{y}{h} + B_{ww}(\Pi)$$

$$\frac{\overline{vv}}{\overline{u}_\tau^2}\left(y^+\right) = B_{vv}(\Pi)$$

- Near to the wall

$$\lim_{y^+\to 0}\frac{\overline{uu}}{\overline{U}^2}\left(y^+\right) = \sigma_{\omega_z}^{+2}$$

$$\lim_{y^+\to 0}\frac{\overline{ww}}{\overline{U}^2}\left(y^+\right) = \sigma_{\omega_x}^{+2}$$

- Modulation of the near wall velocity field in the viscous sublayer by the outer passive eddies (Mathis et al., JFM, 2013)

$$\tau_{0\,p}^{'+}\left(t^+;Re_\tau\right) = \tau_0^{'*+}\left(t^+\right)\left[1 + \beta' u_{OM}^+\left(t^+;Re_\tau\right)\right] + \alpha' u_{OM}^+\left(t^+;Re_\tau\right)$$

- ➔ $\sigma_{\omega_z}^+ \propto \ln\left(Re_\tau\right)$
- ➔ *w structurally similar to u* ➔

$$\sigma_{\omega_x}^+ \propto \ln\left(Re_\tau\right)$$

Intensity of shear layers contributing to the wall normal vorticity

$$\overline{\omega_y^{+2}} = \overline{(\partial u/\partial z)^{+2}} + \overline{(\partial w/\partial x)^{+2}} - 2\overline{(\partial u/\partial z)^{+}(\partial w/\partial x)^{+}}$$

Streamwise $\partial w/\partial x$ layers play a fundamental role in the generation of the quasi-streamwise vortices through the tilting term

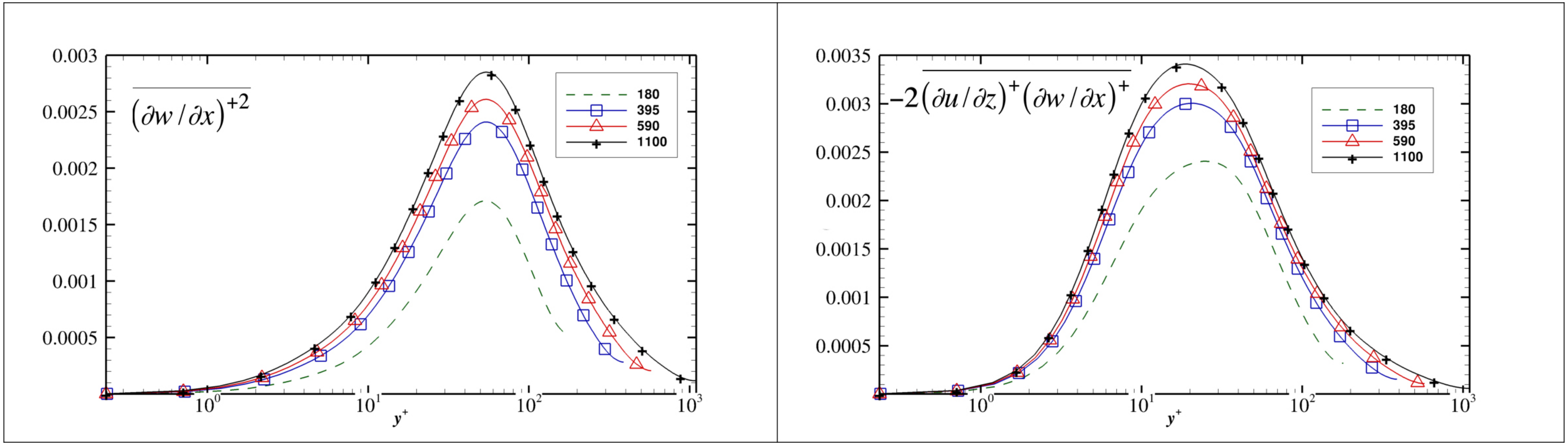
$$\left(-d\overline{U}/dy\right)\left(\partial w/\partial x\right)$$

of the streamwise vorticity transport equation, BUT the contribution of $\overline{(\partial w/\partial x)^{+2}}$ to $\overline{\omega_y^{+2}}$ is ***ONE ORDER*** of magnitude smaller compared to the spanwise shear layers

$$\overline{(\partial u/\partial z)^{+2}}$$

Intensity of shear layers contributing to the wall normal vorticity. Minor contributions coming from *w* streamwise shear layers

Minor contributions strongly Re dependent

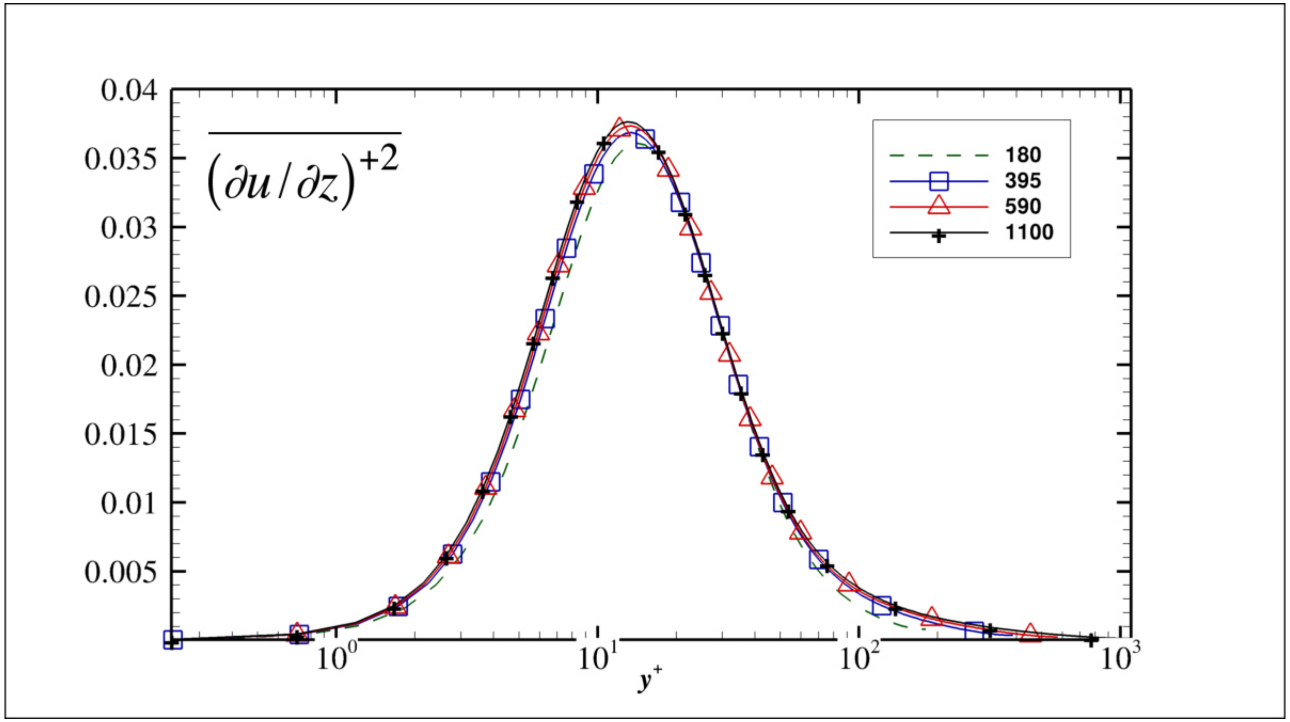


Major Contribution comes from the spanwise *u* shear layers (an order of magnitude)

Remarkably insensitive to Re

Thin and long low and high speed streaks

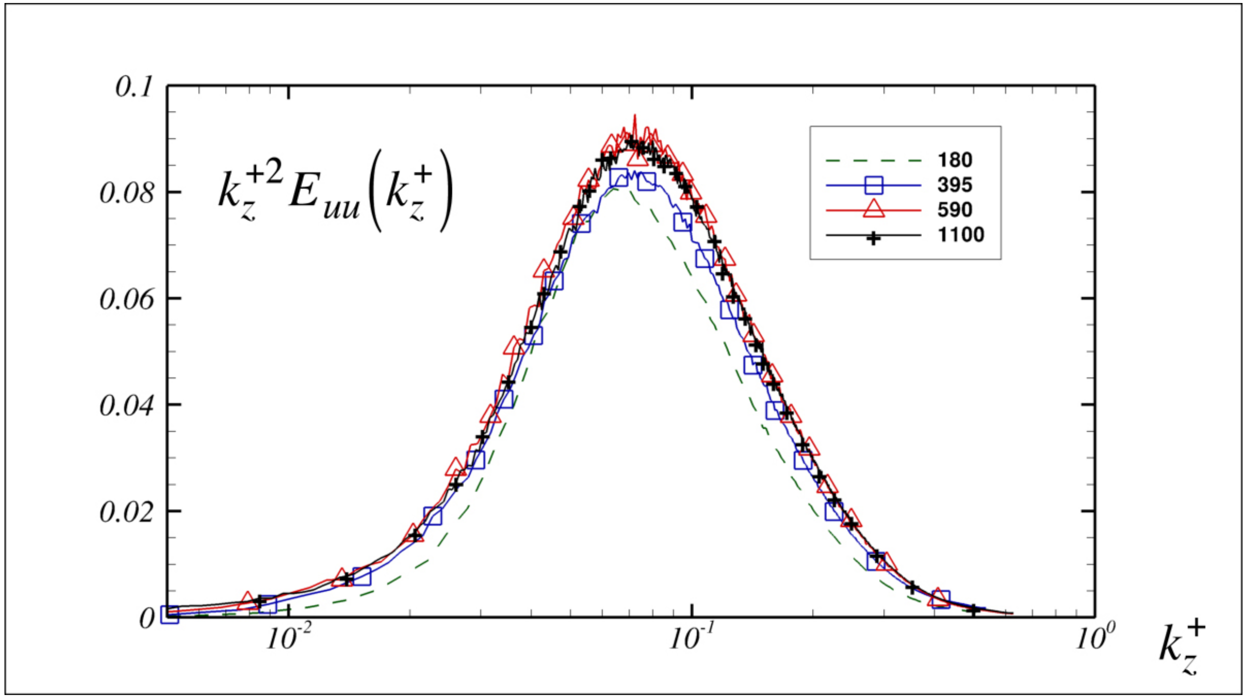
Among all the shear layer components constituting the vorticity components ONLY the intensity $\left(\frac{\partial u}{\partial z}\right)^{+2}$ *IS INSENSITIVE to Re*



One dimensional (spanwise) spectral density
(y^+=15)

$$\overline{\left(\frac{\partial u}{\partial z}\right)^{+2}} = \int_0^\infty k_z^{+2} E^+{}_{uu}\left(k_z^+\right) dk_z^+ \quad \text{==> Reynolds invariant}$$

The one dimensional spectral distribution also is only moderately Re dependent



Vorticity transport

$$\text{Transport}: \frac{\overline{\omega_y^2}}{2}$$

$$0 = P - T - \varepsilon + D$$

Production

$$P = \overline{\omega_y \omega_i \frac{\partial v}{\partial x_i}} = \overline{\omega_y \omega_x \frac{\partial v}{\partial x}} + \overline{\omega_y \omega_y \frac{\partial v}{\partial y}} + \overline{\omega_y \left(\omega_z + \overline{\Omega}_z\right) \frac{\partial v}{\partial z}}$$

Turbulent transport

$$T = \frac{1}{2} \overline{u_i \frac{\partial \omega_y^2}{\partial x_i}} = \frac{1}{2} \overline{\left(u + \overline{U}\right) \frac{\partial \omega_y^2}{\partial x}} + \frac{1}{2} \overline{v \frac{\partial \omega_y^2}{\partial y}} + \frac{1}{2} \overline{w \frac{\partial \omega_y^2}{\partial z}}$$

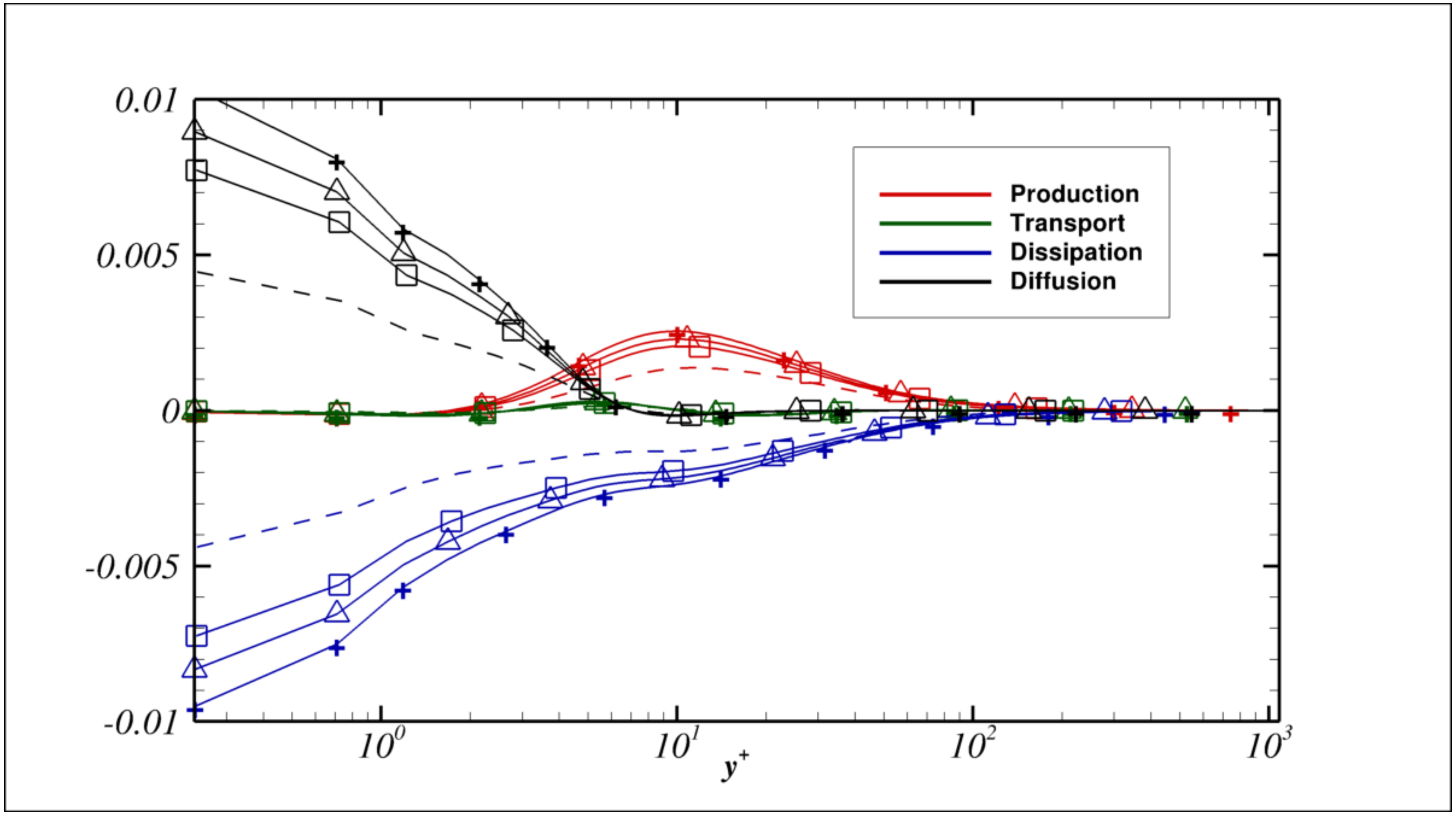
Dissipation

$$\varepsilon = \nu \left\{ \overline{\left(\frac{\partial \omega_y}{\partial x}\right)^2} + \overline{\left(\frac{\partial \omega_y}{\partial y}\right)^2} + \overline{\left(\frac{\partial \omega_y}{\partial z}\right)^2} \right\}$$

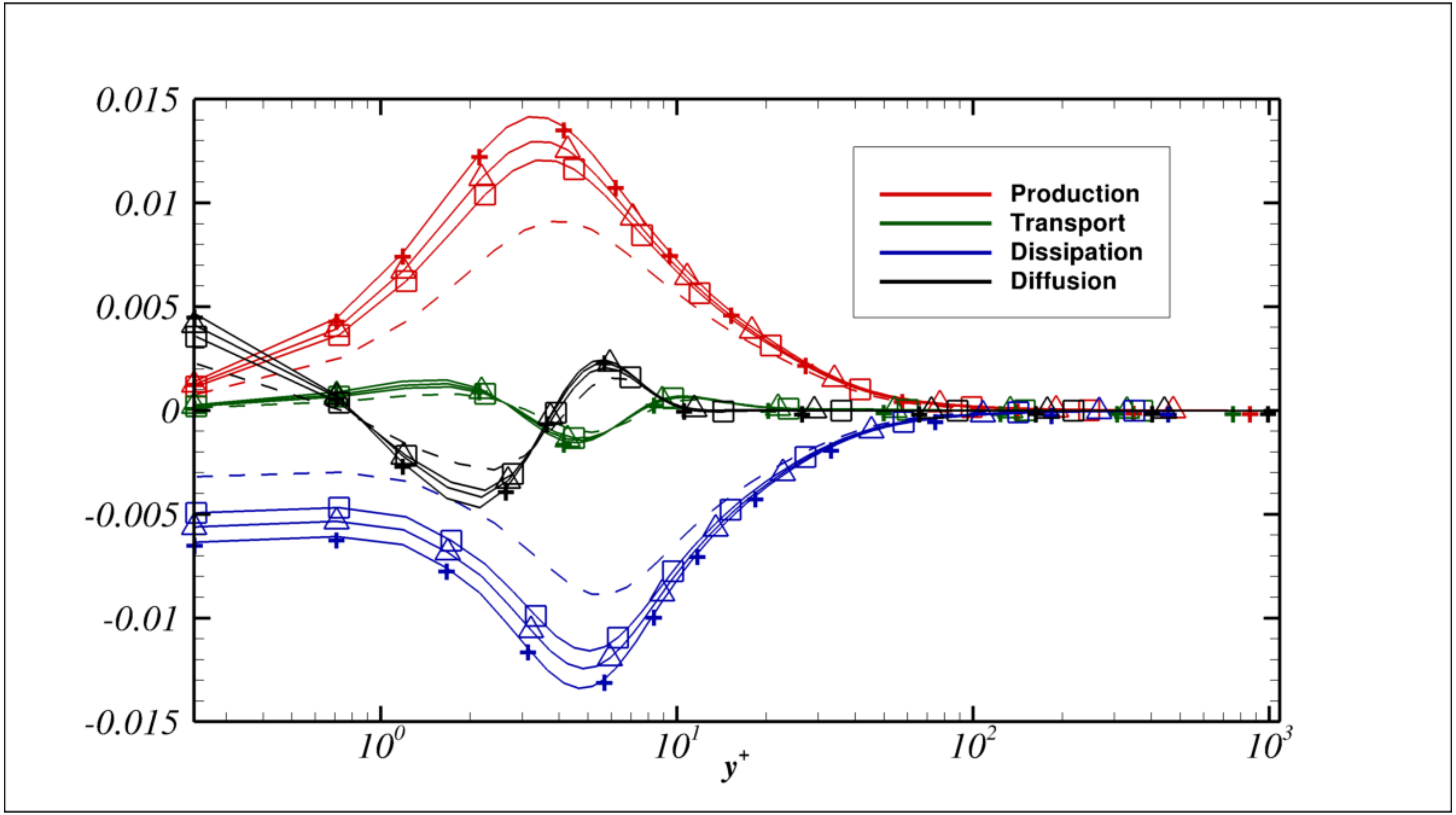
Diffusion

$$D = \nu \frac{1}{2} \overline{\frac{\partial^2 \left(\omega_y\right)^2}{\partial x_i \partial x_i}} = \nu \frac{1}{2} \left\{ \overline{\frac{\partial^2 \left(\omega_y\right)^2}{\partial x^2}} + \overline{\frac{\partial^2 \left(\omega_y\right)^2}{\partial y^2}} + \overline{\frac{\partial^2 \left(\omega_y\right)^2}{\partial z^2}} \right\}$$

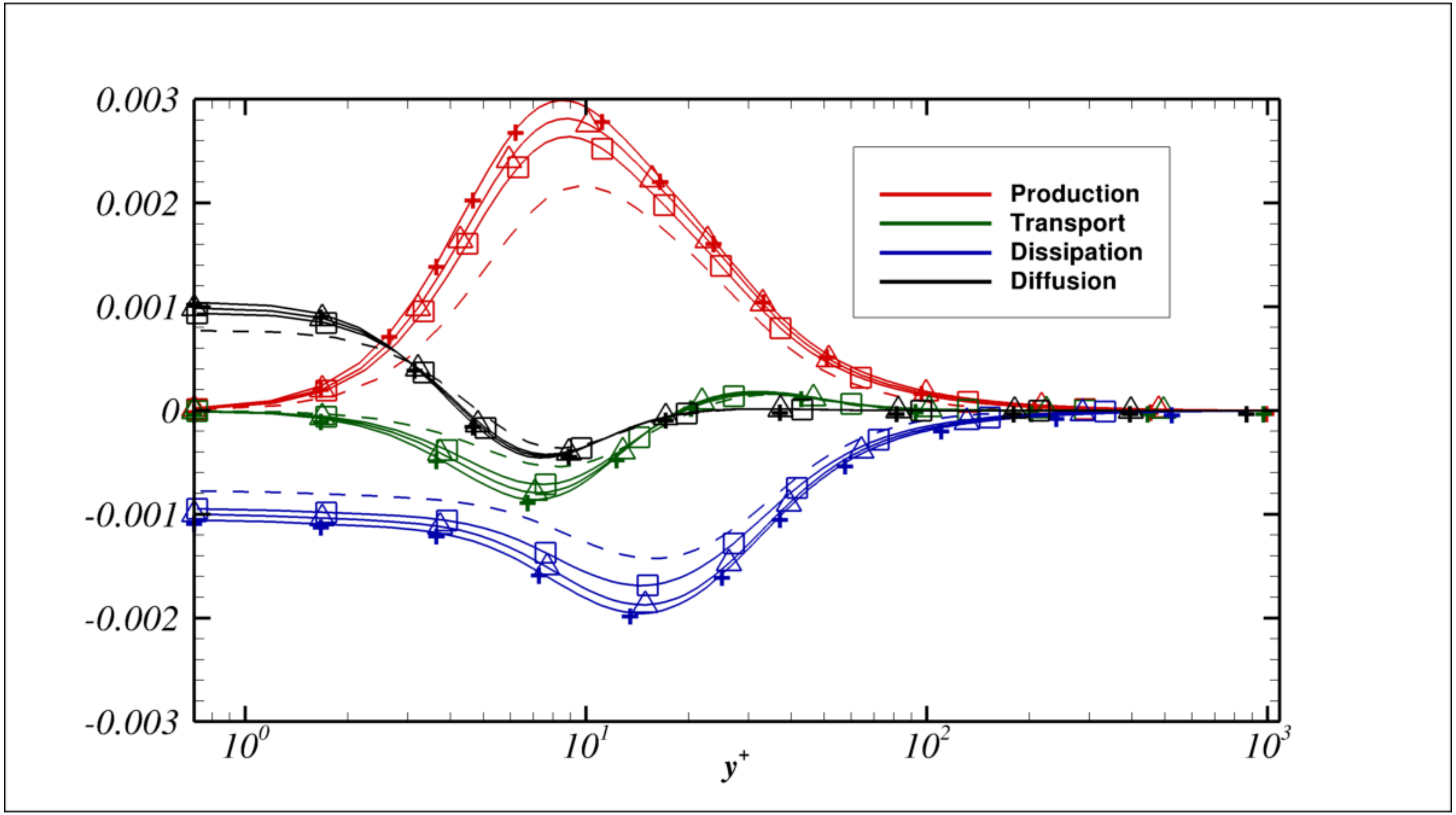
Streamwise vorticity (intensity) transport



Spanwise vorticity transport



Wall normal vorticity transport



Vorticity components transport General remarks

- Maximum production of the streamwise and wall normal vorticity intensities takes place in the median buffer layer at y^+=10 to 15.

**Despite the fact that $\overline{\omega_y^{+2}}$ is Re independent, the production and dissipation of the wall normal vorticity depends on the Reynolds number. The dissipation is not in equilibrium with the production.

- On the contrary the spanwise vorticity production peaks in the viscous sublayer at y^+=3.
- Dissipation is in equilibrium with diffusion next to the wall as usual. Both quantities are Re independent near y=0 in $\overline{\omega_y^{+2}}$.

Spanwise *u* shear layers Transport

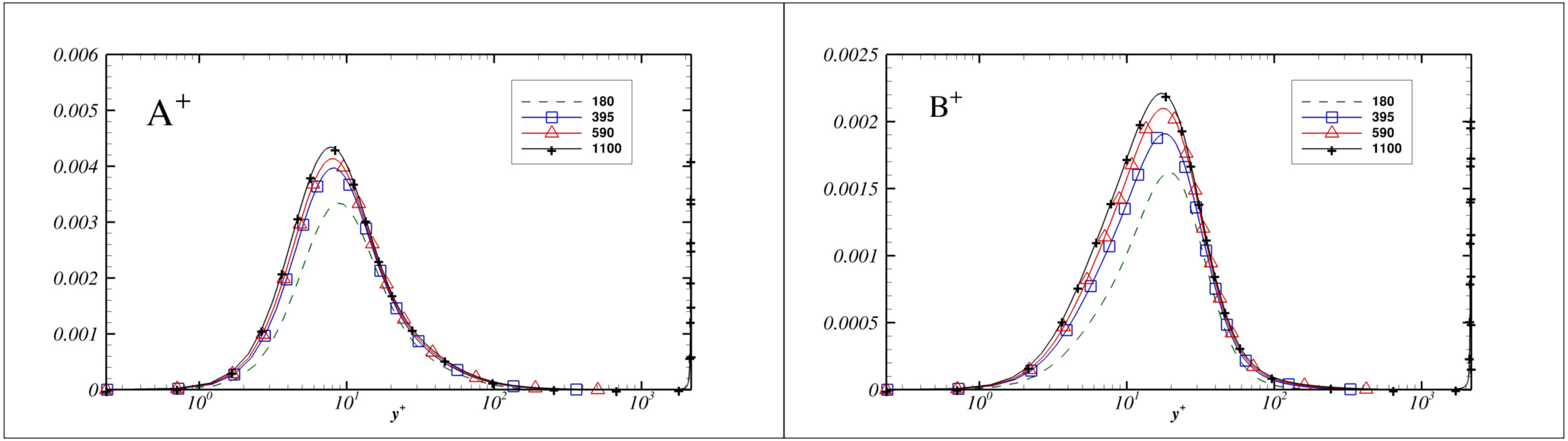
- $$\frac{D}{Dt}\left(\frac{\partial u}{\partial z}\right)^2 = -2\overline{\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}\frac{\partial(\overline{U}+u)}{\partial y}} + 2\overline{\left(\frac{\partial u}{\partial z}\right)^2\frac{\partial v}{\partial y}} + 2\overline{R\frac{\partial u}{\partial z}}$$

A B

$$R = -\frac{\partial^2 p}{\partial x \partial z} + \nu\nabla^2\left(\frac{\partial u}{\partial z}\right)$$

- A is a kind of production by tilting of the mean shear

Spanwise *u* shear layers
Transport



A$^+$ is twice larger than B$^+$

Both of them are remarkably Re independent at $y^+ > 20$

Discussion

WHY $\sigma^+_{\omega_y}$ is invariant with Re?

- One has in Fourier space

$$\underline{\underline{u}} = -\frac{\iota k_z}{k_x^2 + k_z^2}\underline{\underline{\omega_y}} + \frac{\iota k_x}{k_x^2 + k_z^2}\underline{\underline{\left(\frac{\partial v}{\partial y}\right)}} \quad \text{(Definition + Continuity)}$$

$$E_{uu} = \frac{k_z^2}{\left(k_x^2 + k_z^2\right)^2} E_{\omega_y\omega_y} + \frac{k_x^2}{\left(k_x^2 + k_z^2\right)^2} E_{\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}} - \frac{2k_x k_z}{\left(k_x^2 + k_z^2\right)^2}\left\{ I_{\omega_y} I_{\frac{\partial v}{\partial y}} + R_{\omega_y} R_{\frac{\partial v}{\partial y}} \right\}$$

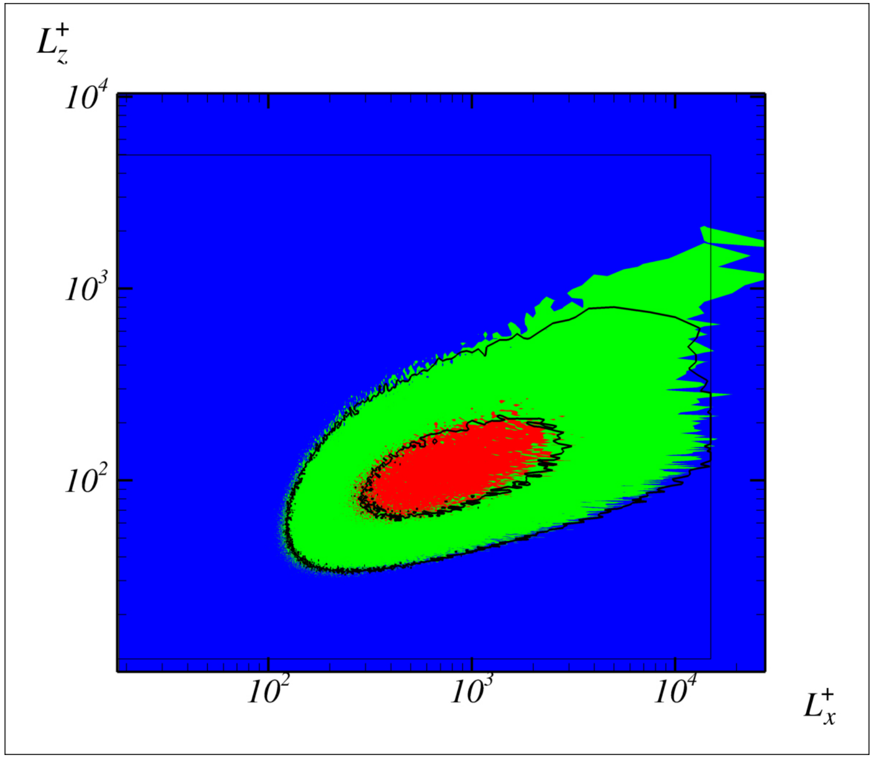
I : Imaginary part, R : Real part ; Premultiplied spectra of each term in wall units $\Rightarrow$

$$\mathrm{A}_{\omega_y} = k_x k_z \frac{k_z^2}{\left(k_x^2 + k_z^2\right)^2} E_{\omega_y\omega_y} \qquad B_{\frac{\partial v}{\partial y}} = k_x k_z \frac{k_x^2}{\left(k_x^2 + k_z^2\right)^2} E_{\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}}$$

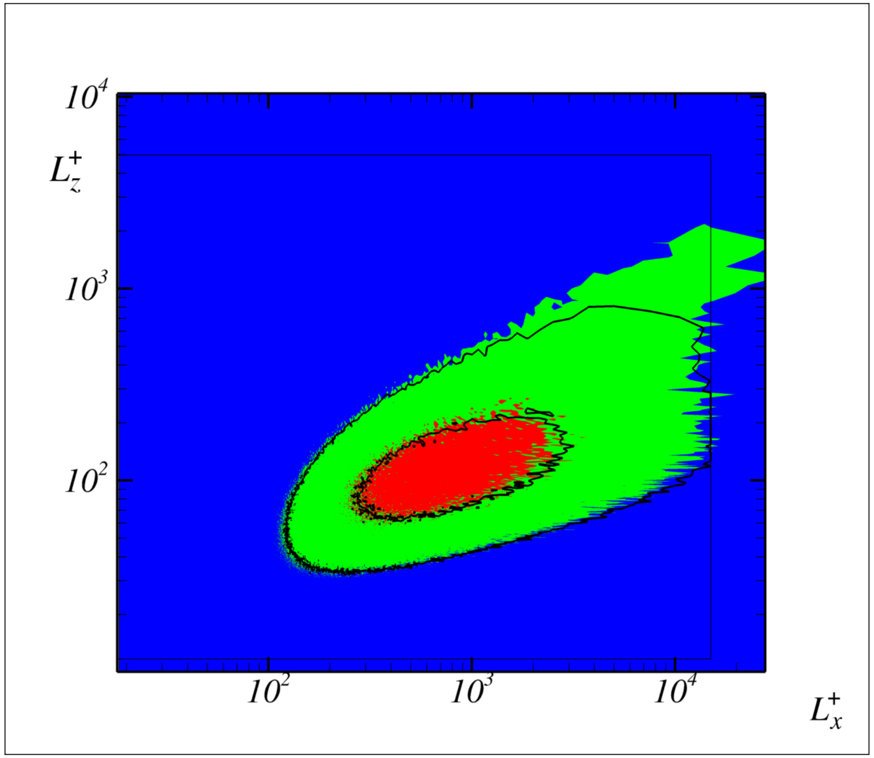
$$C_{\omega_y;\frac{\partial v}{\partial y}} = -\frac{2k_x^2 k_z^2}{\left(k_x^2 + k_z^2\right)^2}\left\{ I_{\omega_y} I_{\frac{\partial v}{\partial y}} + R_{\omega_y} R_{\frac{\partial v}{\partial y}} \right\} \qquad D = B + C$$

Premultiplied spd's of the ensemble of these terms

$$k_x k_z E_{uu}$$



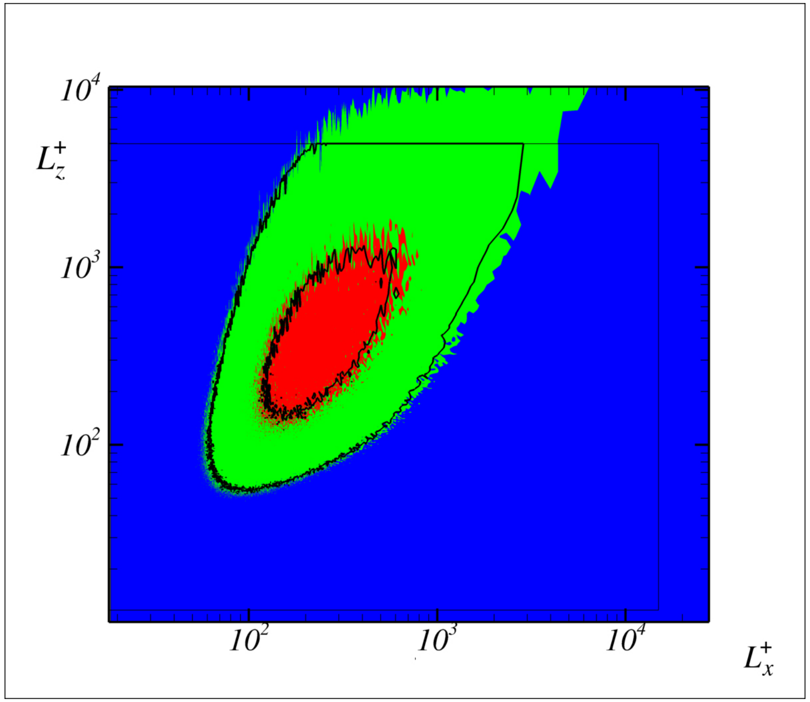
$$A_{\omega_y} = k_x k_z \frac{k_z^2}{\left(k_x^2 + k_z^2\right)^2} E_{\omega_y \omega_y}$$



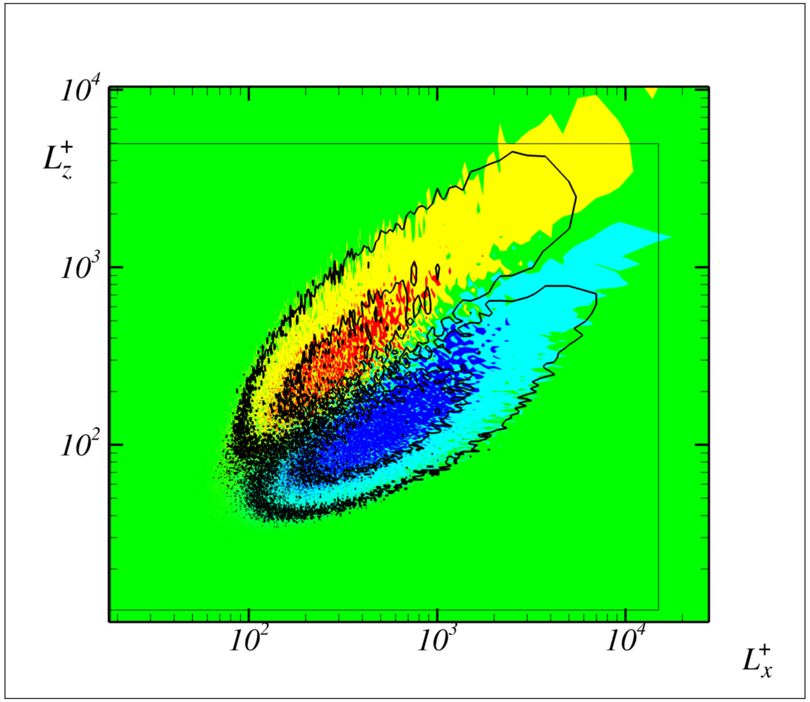
Reto 395(lines) and 1100 (color). Two contours for each spectral density , 0.125 and 0. 625 the maximum at the highest Re number (as in Hoyas and Jiménez, PoF, 2006). Note the inactive ridge. ***The contours in the spectra related to the wall normal vorticity have almost the same magnitude.***

Terms coming from the flux of the wall normal velocity and interactions between v and wall normal velocity have different spectral supports and ***are an order of magnitude smaller***

$$k_x k_z \frac{k_x^2}{\left(k_x^2+k_z^2\right)^2} E_{\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}}$$



$$-\frac{2k_x^2 k_z^2}{\left(k_x^2+k_z^2\right)^2}\left\{I_{\omega_y} I_{\frac{\partial v}{\partial y}} + R_{\omega_y} R_{\frac{\partial v}{\partial y}}\right\}$$



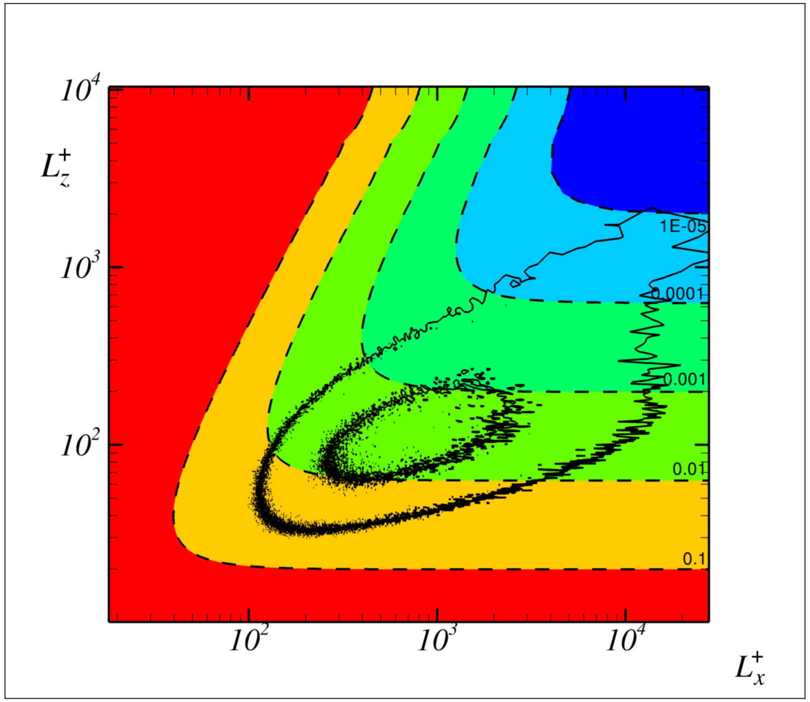
In yellow: positive contours
In blue: Negative contours

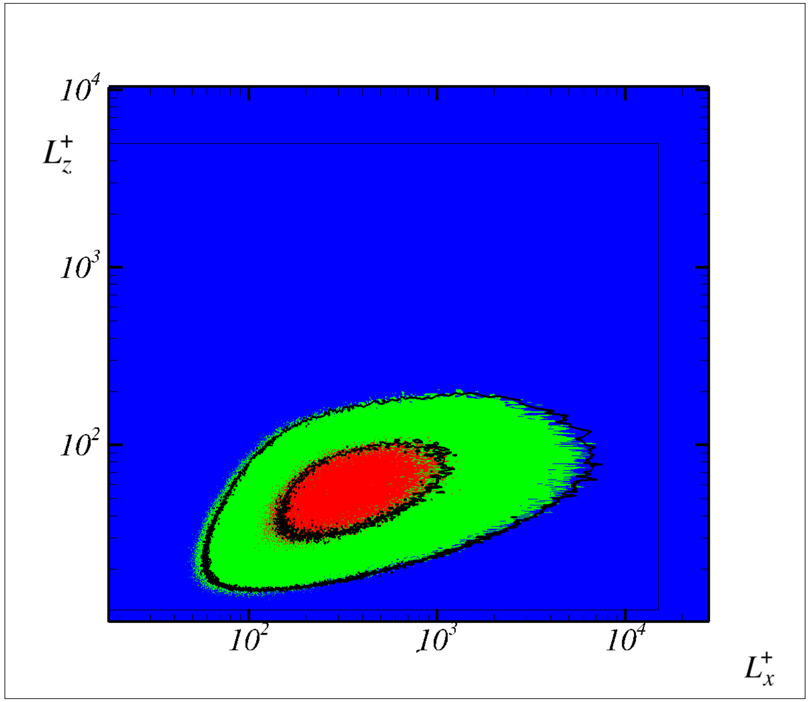
➔ *Wall normal vorticity is connected to the low pass filtered u*

$$E_{uu} \approx \frac{k_z^2}{\left(k_x^2+k_z^2\right)^2} E_{\omega_y\omega_y} \Rightarrow \underline{\underline{u}} \equiv -\frac{\iota k_z}{k_x^2+k_z^2} \underline{\underline{\omega_y}} \Rightarrow \omega_y \equiv \left[-\iota \frac{k_x^2+k_z^2}{k_z}\right] \underline{\underline{u}}$$

LOW PASS FILTER $H(\iota k_x, \iota k_z)$

$k_x k_z E_{uu}$ and contours of the filter amplitude $|H|^2$ $\Rightarrow$ Filters ridges $k_x k_z E_{\omega_y\omega_y}$ INDEPENDENT of Re_τ





CONCLUSION

- Maximum production of the streamwise and wall normal vorticity intensities takes place in the median buffer layer at $y^+=15$.
- On the contrary the spanwise vorticity production peaks in the viscous sublayer at $y^+=3$.
- Wall normal vorticity is dominated by the spanwise u velocity shear layers whose intensity is Re independent.
- It is related to the low pass filtered streamwise velocity field and therefore not influenced by the passive structures. Its intensity distribution in wall units is universal.

- ?