Flow regimes and vorticity dynamics in T-mixers

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ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

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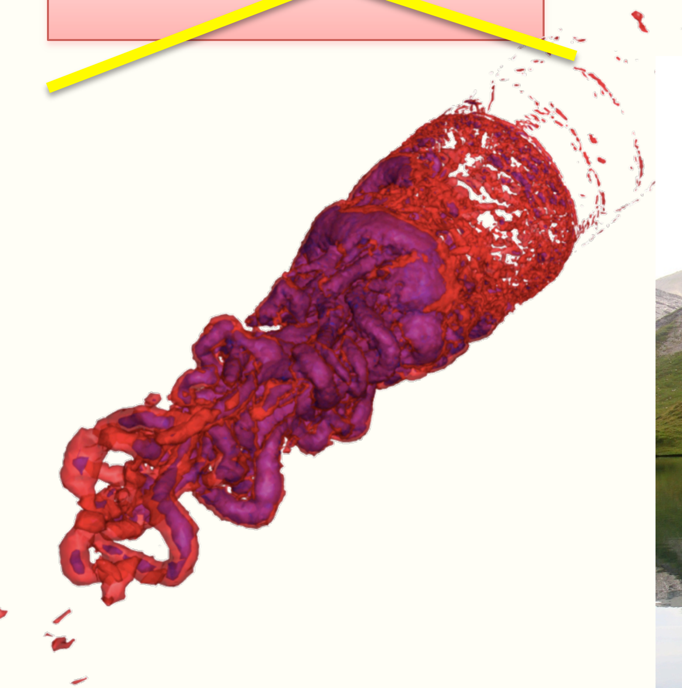
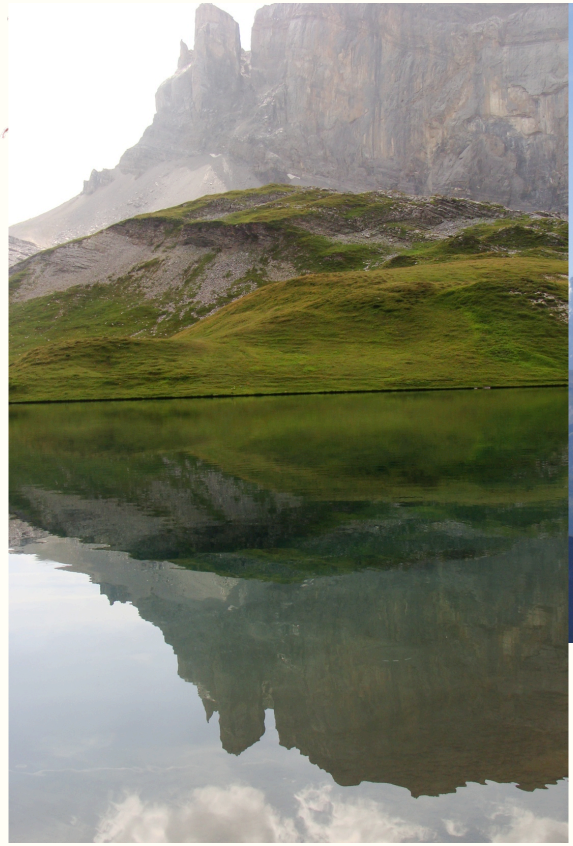
Shared passions with Paolo Orlandi

~~Turbulence~~

~~Mountains~~





Vorticity dynamics

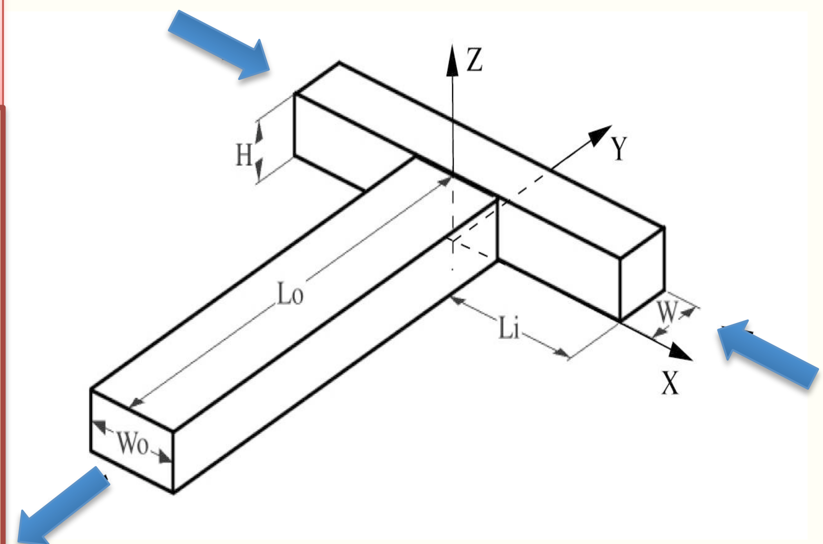
Micro T-mixers - Motivation

- ✓ T-mixers are important devices in microfluidics (e.g. junction elements in complex micro systems).
- ✓ They are aimed at promoting mixing between two fluid streams.
- ✓ Low Reynolds numbers (because of the small dimensions).

Different flow regimes with increasing Reynolds numbers

Reynolds number

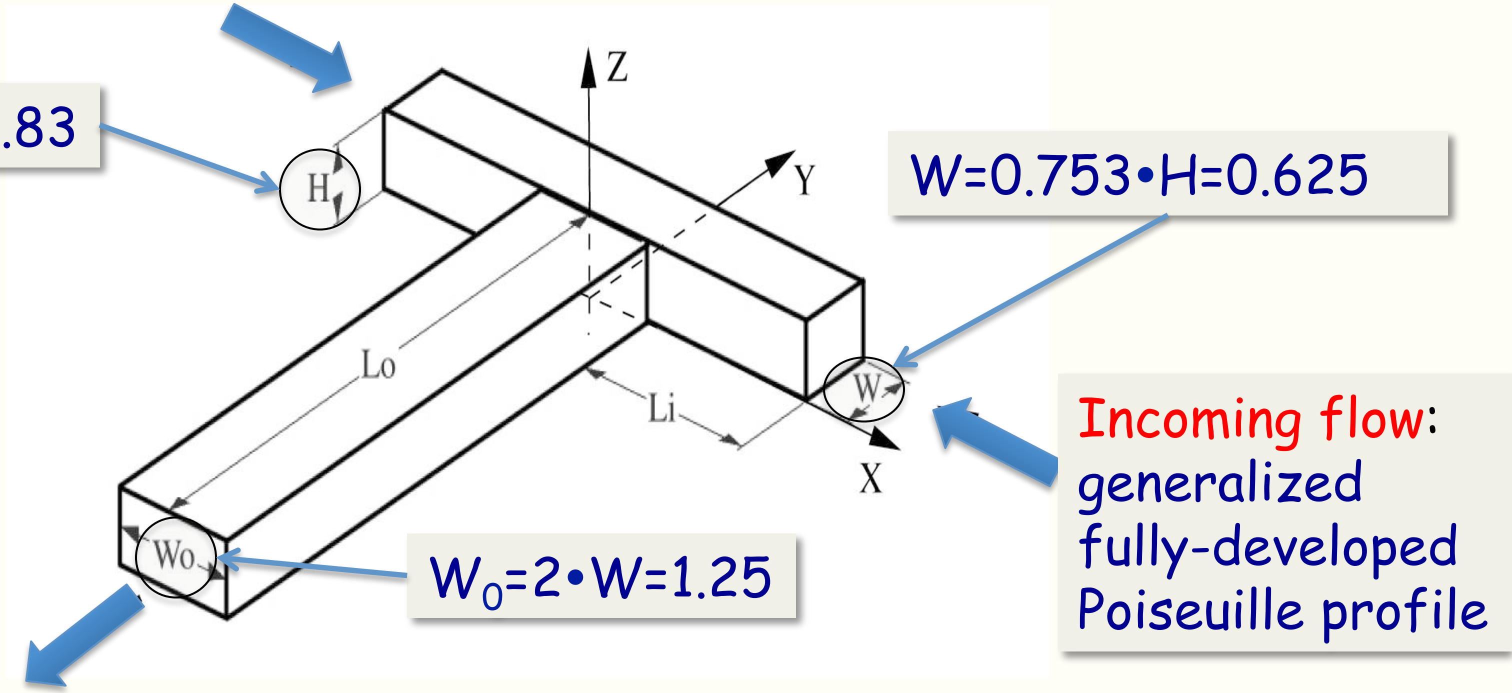
- ☹ Stratified regime: the two streams remain segregated; mixing only due to diffusion).
- ☹ Vortex regime: a double pair of steady counter-rotating vortices in the mixing channel. The streams remain segregated.
- ☺ Engulfment regime: steady and asymmetric. One stream reaches the opposite side of the mixing channel.
- ? Periodic unsteady regimes.
- ☺ Turbulent "chaotic" unsteady regime.



Our work:

- ✓ Analysis of the engulfment and periodic unsteady regimes through **DNS**. Focus on 3D vortical structures.
- ✓ **Linear stability** and **sensitivity analyses** to accurately estimate the critical conditions for the onset of the different regimes and to obtain information on possible control strategies.

Flow configuration and methodology



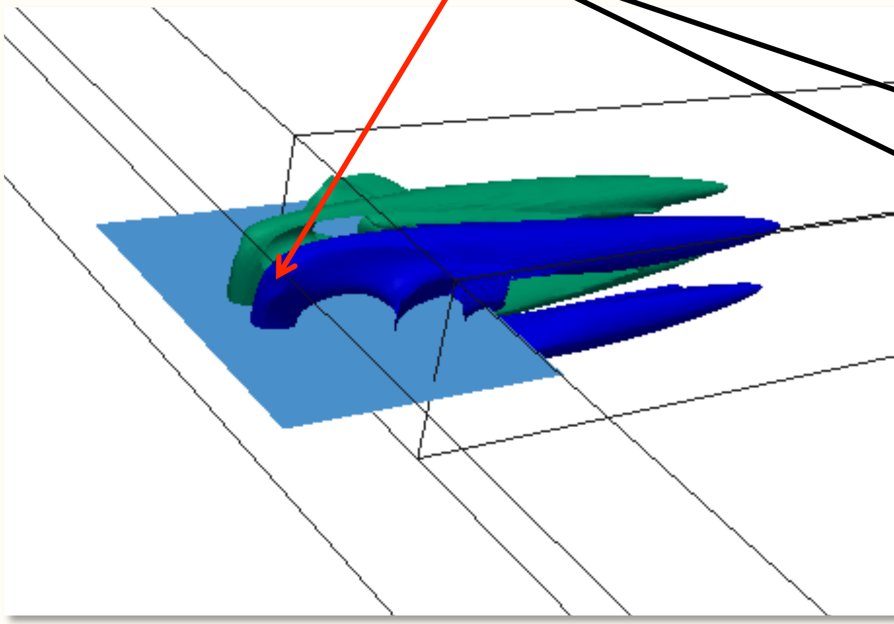
- ✓ Reference Length: the idraulic diameter of the outflow pipe Dh
- ✓ Reference Velocity: bulk velocity at the inlet pipes

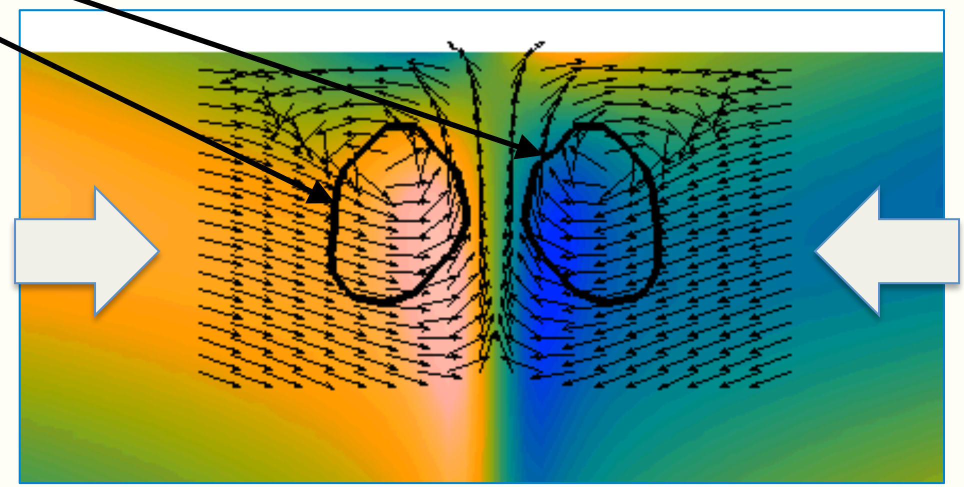
- ✓ DNS were carried out using NEK5000 (open-source spectral element code):
  - ☞ N-th order Lagrangian polynomial interpolants in each grid element (N-2 for pressure).
  - ☞ Third-order backward finite difference scheme for time advancing.
- ✓ Example of grid size/resolution:
  - ☞ steady engulfment: N=11 and 7000 elements → 9.4×10^6 d.o.f. for velocity
  - ☞ unsteady regimes: N=9 and 1568 elements → 5.2×10^6 d.o.f. for velocity.

Vortex regime (DNS at Re=140)

Vortical structures identified by the λ_2 criterion

In-plane velocity vectors and (black thick line) trace of the vortices on the cutting plane

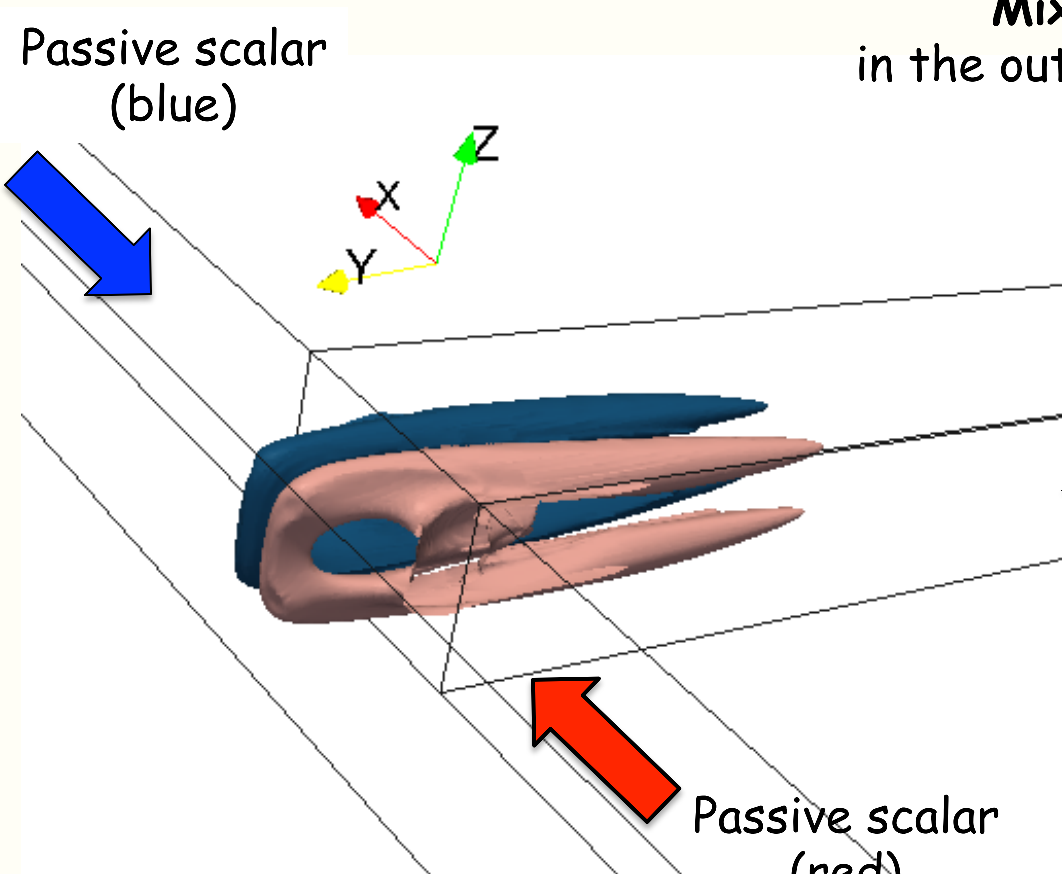




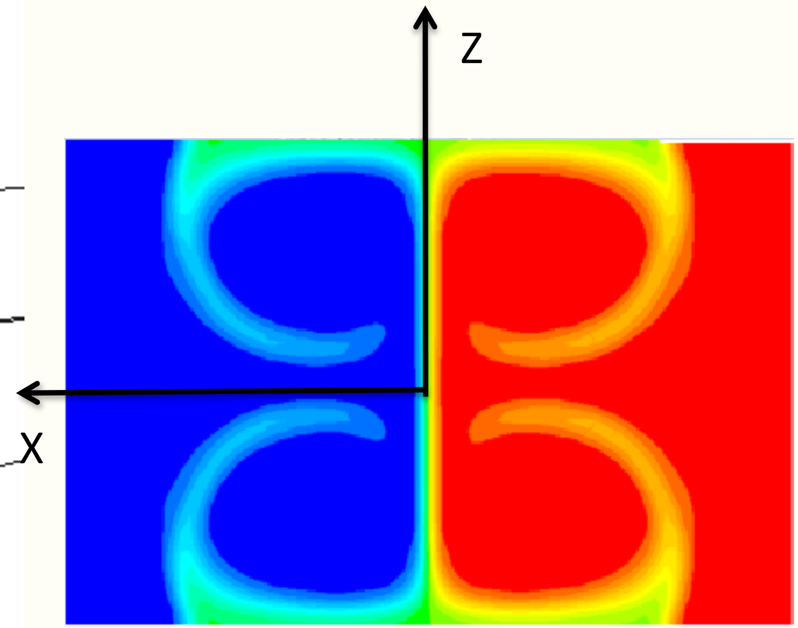
- ✓ The two counter-rotating vortical structures are the skeleton of the flow.
- ✓ These two structures collect the vorticity of the separation region near the top wall of the mixer and they are convected in the outflow pipe by the flow
- ✓ The two reflectional symmetries of the geometry are preserved in the flow → low mixing (only by means of diffusion).

See Fani et al. PoF, 25(6), 2013

Vortex regime (DNS at Re=140)



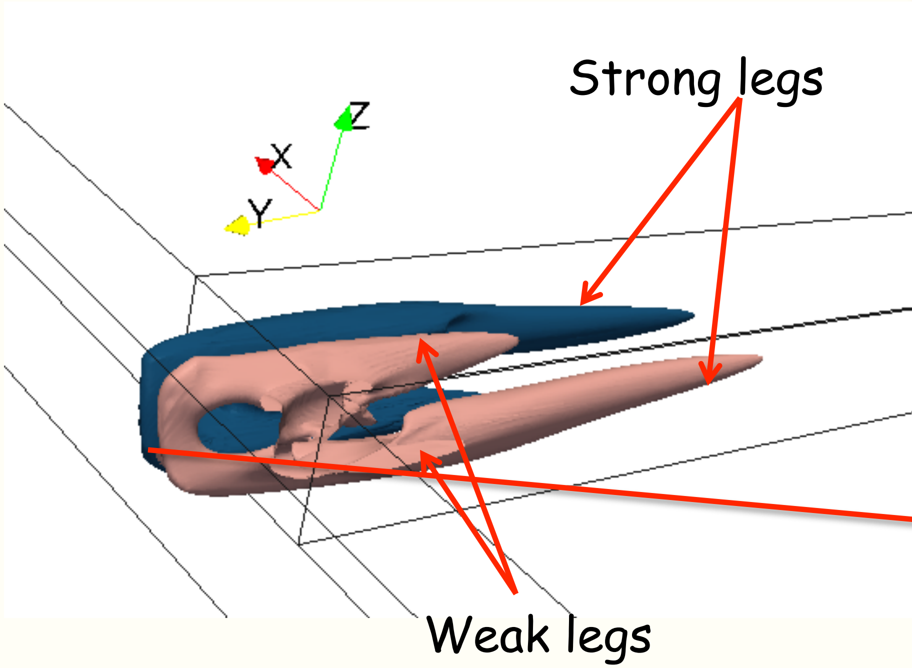
Mixing properties (passive scalar) in the outflow channel: **mainly diffusive (poor)**



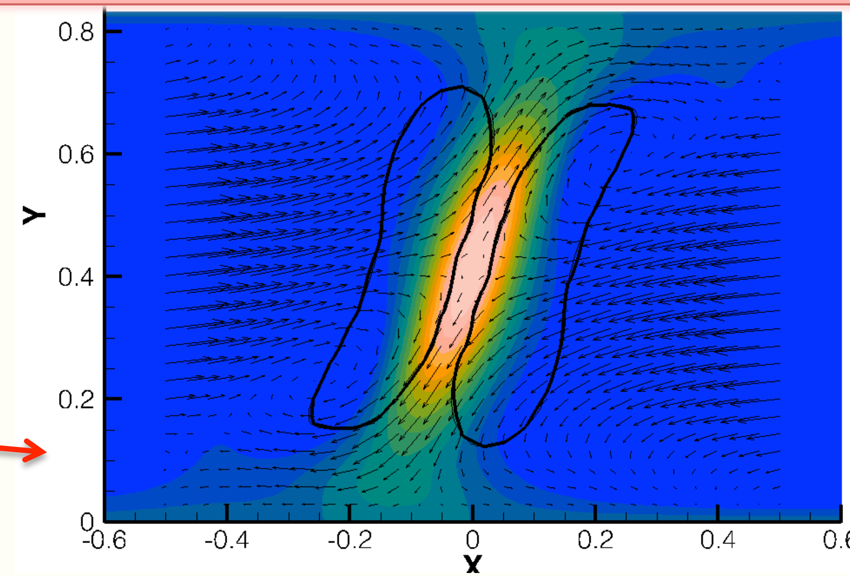
See Fani et al. PoF, 25(6), 2013

Engulfment regime (DNS at Re=160)

Vortical structures identified by the λ_2 criterion



In-plane velocity vectors and (black thick line) trace of the vortices on the cutting plane (top view)

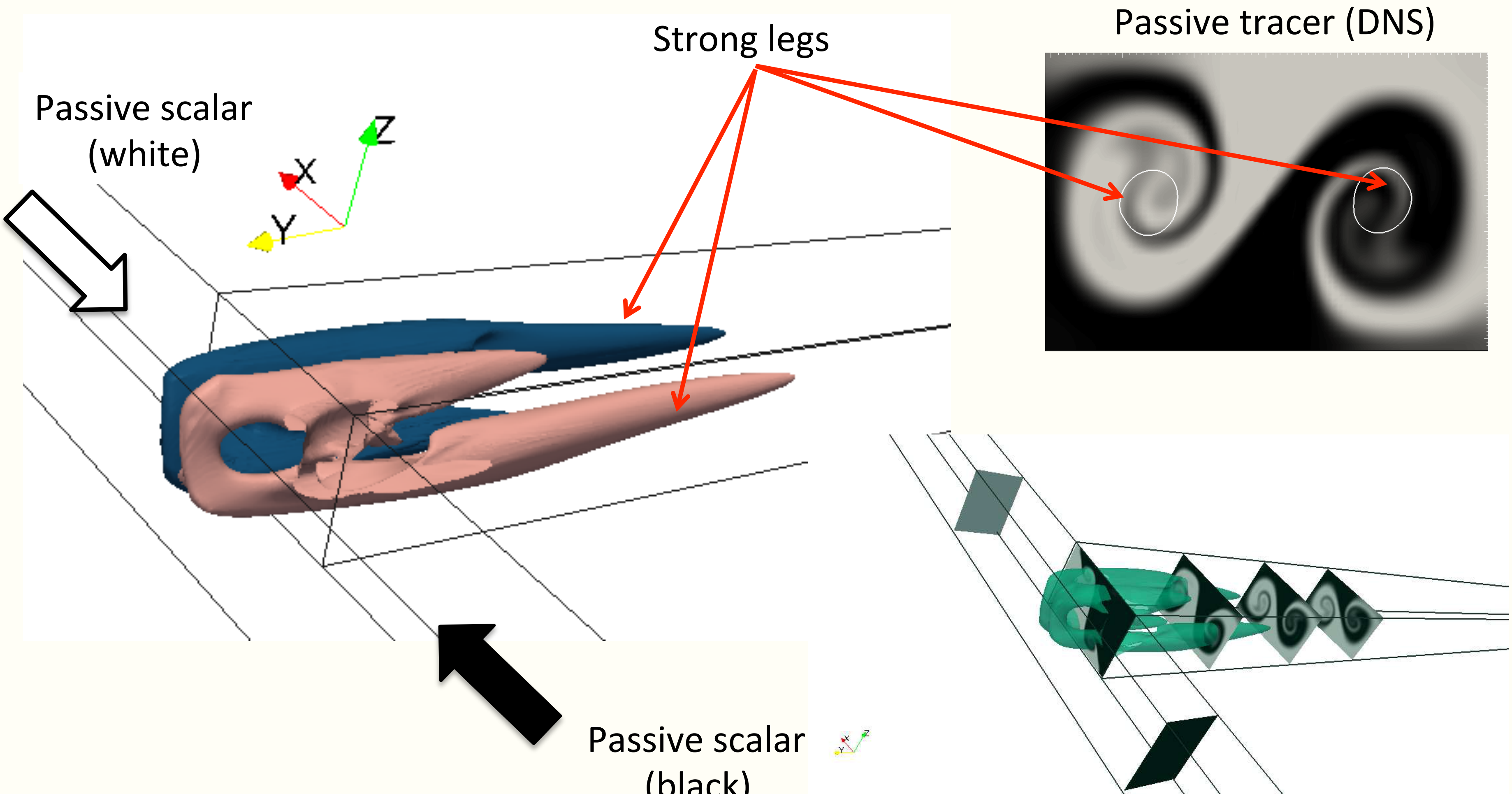


- ✓ The two vortices at the confluence are characterized by a tilt angle and symmetry is lost.
- ✓ As a consequence, the two legs of each vortex entering in the mixing channel are not equal in terms of intensity, shape and position.

See Fani et al. PoF, 25(6), 2013

Engulfment regime (DNS at Re=160)

- ✓ Moving towards the end of the outflow channel, only the strongest legs survive → only a couple of co-rotating vortices can be observed far enough from the T-junction, which induce velocities strongly enhancing mixing between the two streams.

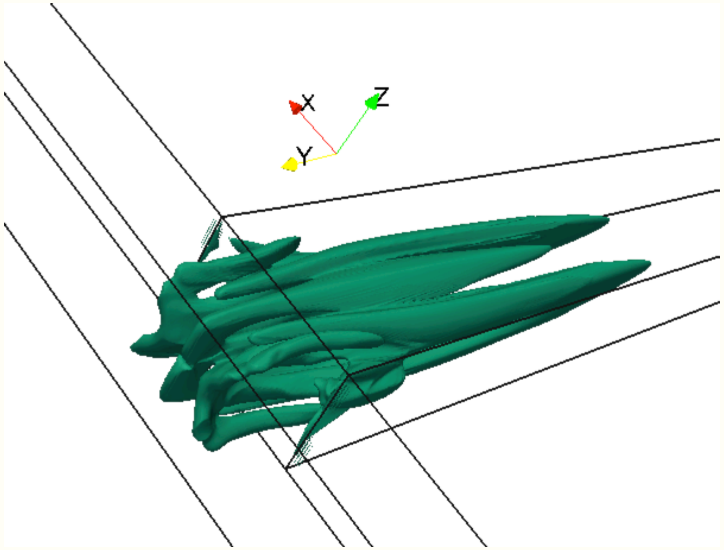


See Fani et al. PoF, 25(6), 2013

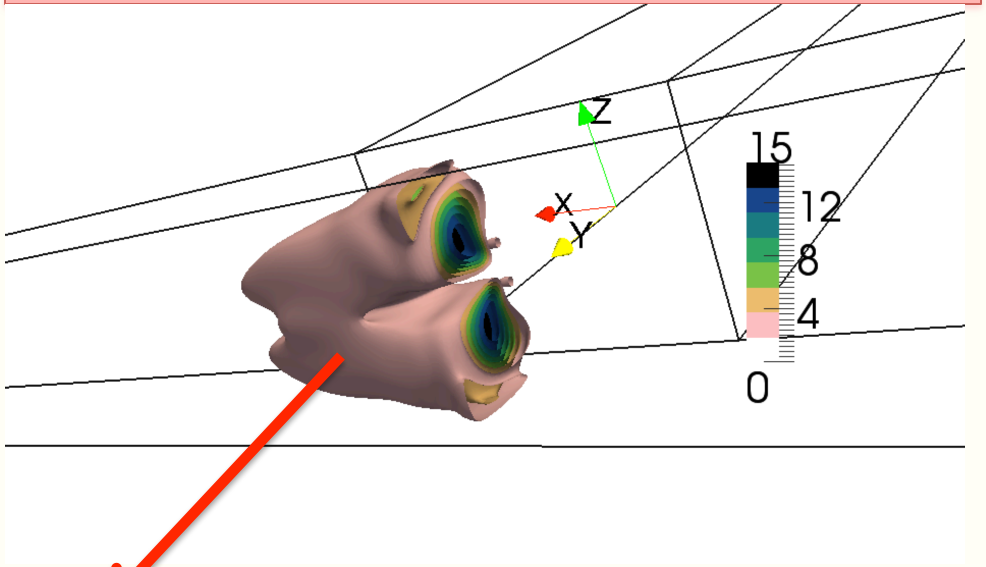
Engulfment regime – Stability analysis

- ✓ Critical Re≈140 in agreement with DNS and literature
- ✓ Onset of the engulfment instability is due to a pitchfork bifurcation → real-valued unstable eigenvalue and eigenvector.

Vortical structures of the global **unstable mode**



Instability core (overlap between the direct and adjoint modes
(Giannetti & Luchini, JFM 581, 2007)



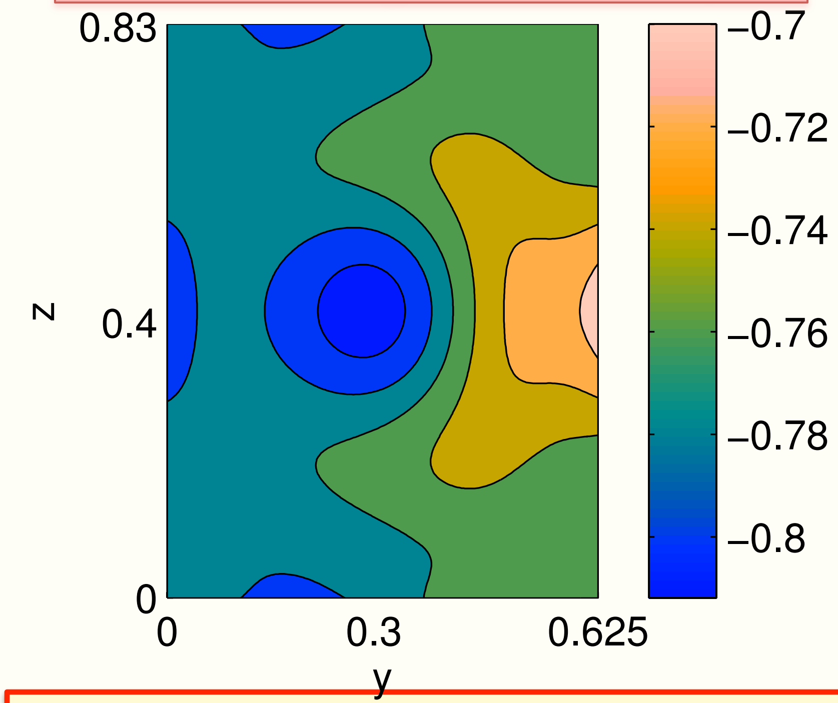
- ✓ The instability core is localized at the intersection of the channels, and, in particular, inside the recirculation zones where the 3D counter-rotating vortical structures originate.

For details on stability analysis methods and results see Fani et al. PoF, 25(6), 2013

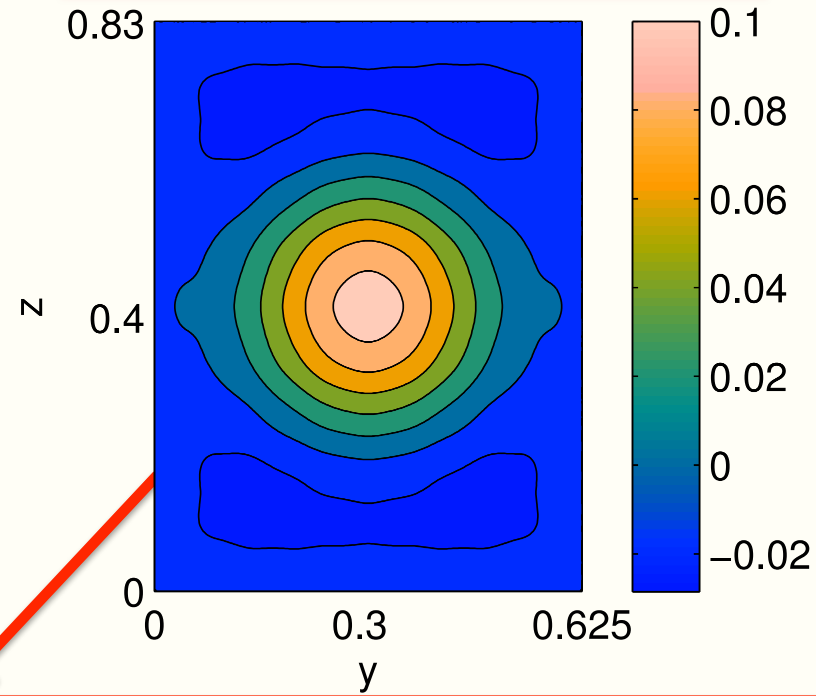
Engulfment regime – Sensitivity analysis

- ✓ How the inflow velocity profile influences the onset of the engulfment regime
- ✓ Sensitivity to a perturbation computed as the difference between a generalized Poiseuille profile and a not fully developed one (more flat).

Sensitivity map (related to the unstable eigenmode shift)



Velocity perturbation



- ✓ The onset of the engulfment regime is delayed if the inlet velocity profile is not fully developed (short inlet channels).
- ✓ This was confirmed by DNS

For details on stability analysis methods and results see Fani et al. PoF, 25(6), 2013

Further increasing Re: unsteady regimes

Experiments and images from:

S. Thomas and T. a. Ameel, Exp. Fluids, vol. 49(6)2010.

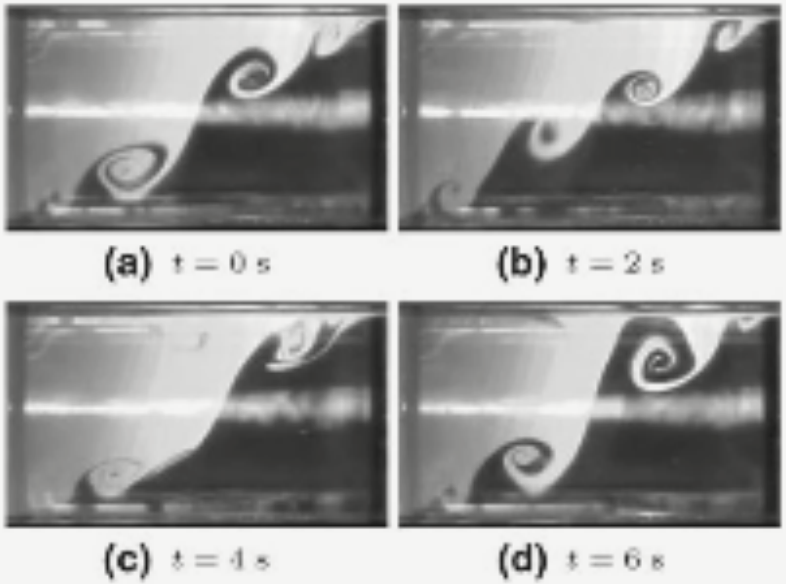
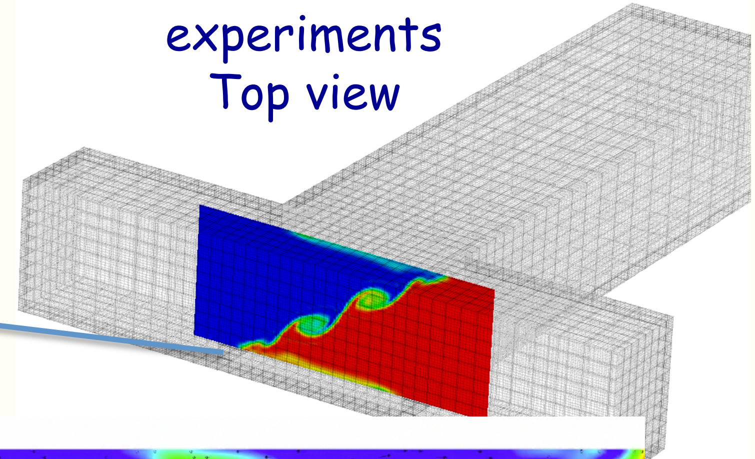
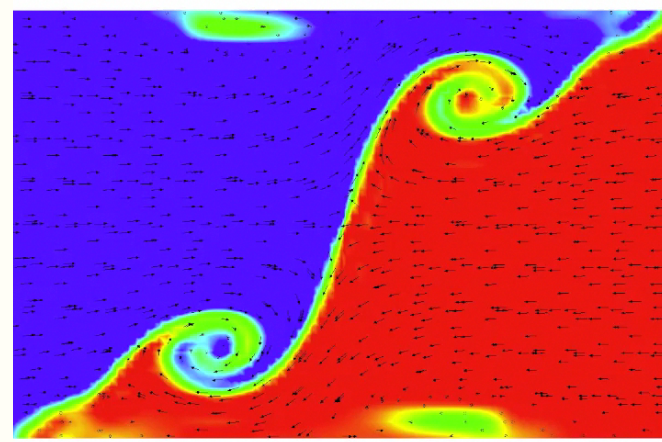


Fig. 8 LIF still frames extracted in two-second intervals for unsteady asymmetric flow in the junction centerplane ($x = 0$) for $Re = 290$

DNS at Re=240; tracer as in experiments
Top view

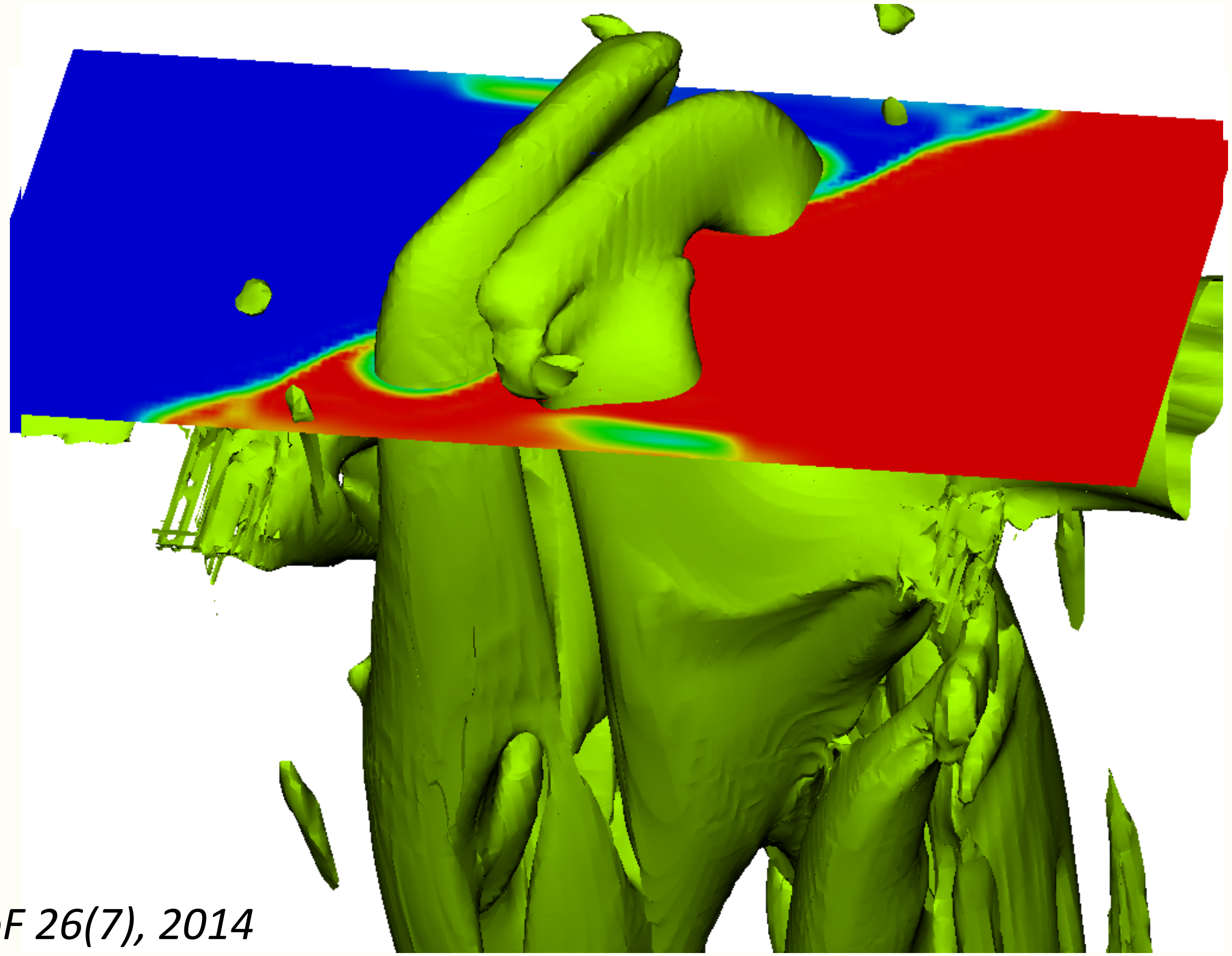




- ✓ When the Reynolds number is further increased above a second critical value, there is experimental evidence in the literature of an unsteady periodic regime.
- ✓ "Kelvin-Helmotz like" vortical structures are observed in the top part of the mixer.

Unsteady asymmetric regime (DNS)

These "Kelvin-Helmotz like" vortices are actually **traces of the strong legs** of the two vortical structures typical of the steady engulfment → the unsteady flow behavior is related to the dynamics of these structures



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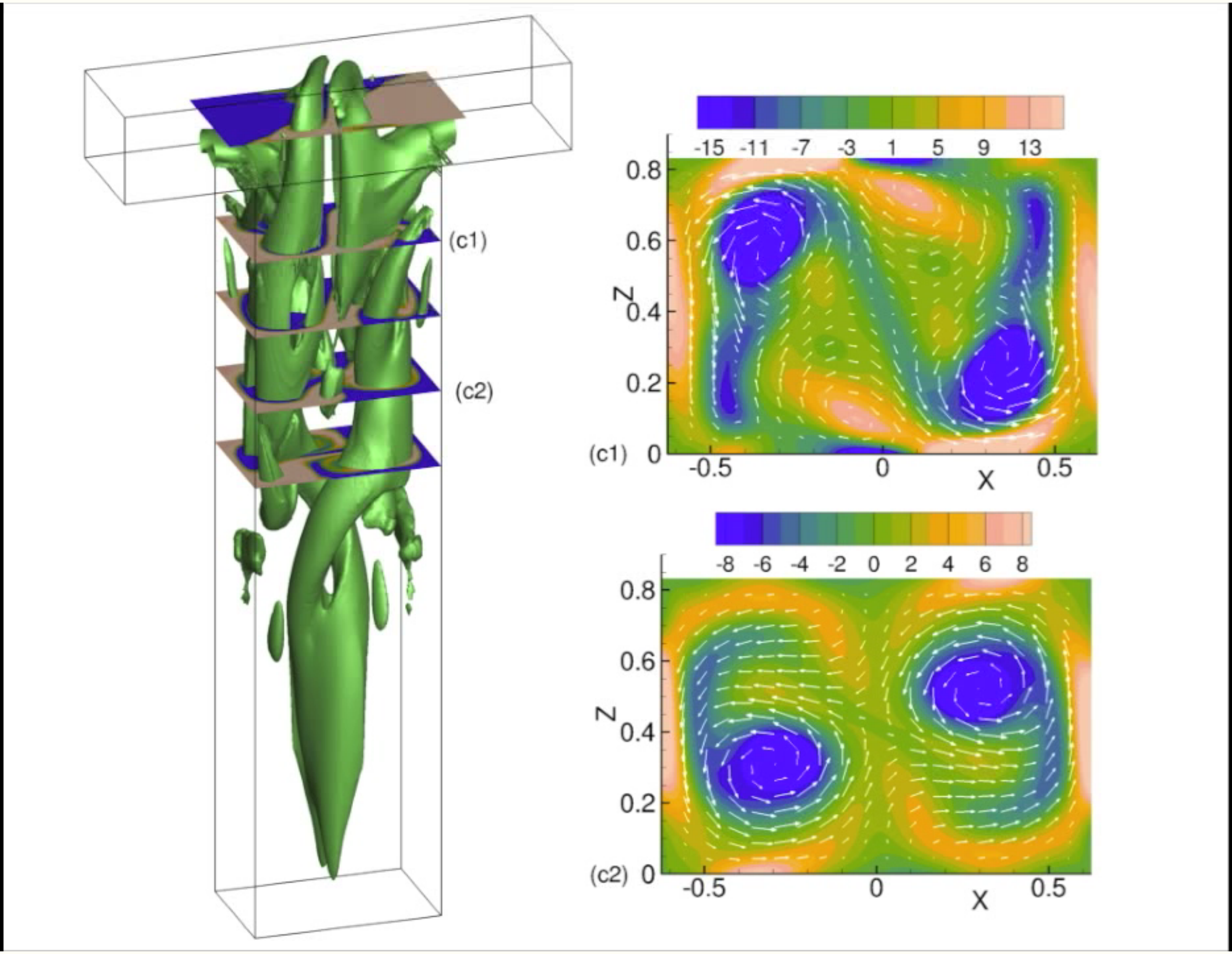
Unsteady asymmetric regime (DNS)



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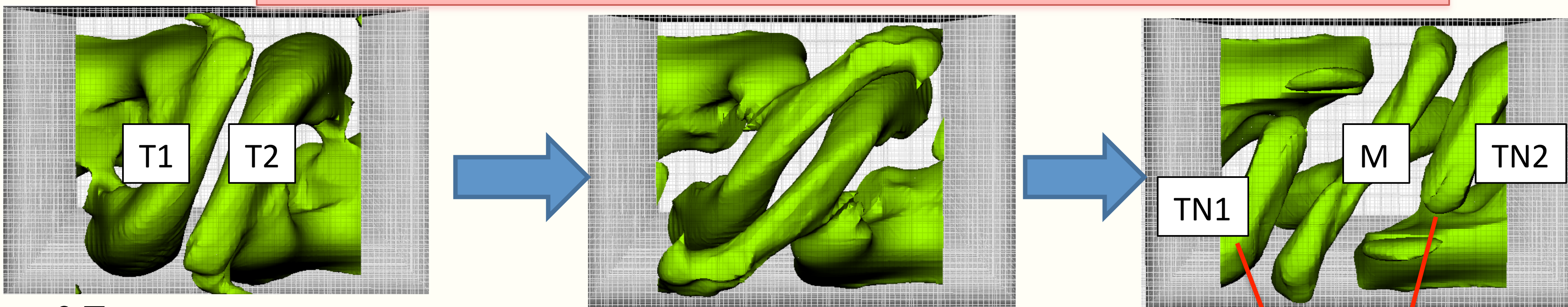
Unsteady asymmetric regime (DNS)

Dynamics of the top part of the 3D vortical structure



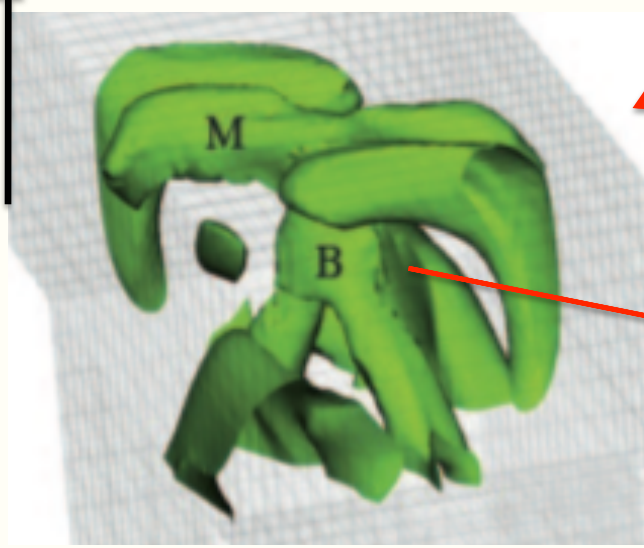
$t = 0T$ $t = 0.45T$ $t = 0.65T$

Tilting angle increases up to a maximum value

T1 and **T2** form **M** which disappears (vorticity of opposite sign)

Formation of of a new separation zone near the top wall which feeds two new 3D vortical structures (TN1 and TN2)

Y-direction

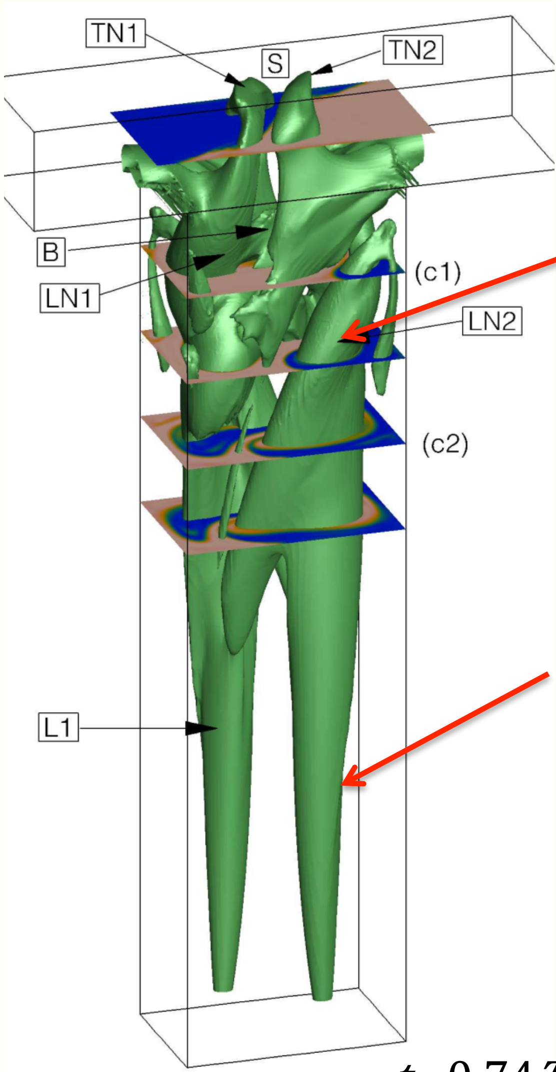


Blob of y-vorticity B convected in the mixing channel

See Fani et al. PoF 26(7), 2014

Unsteady asymmetric regime (DNS)

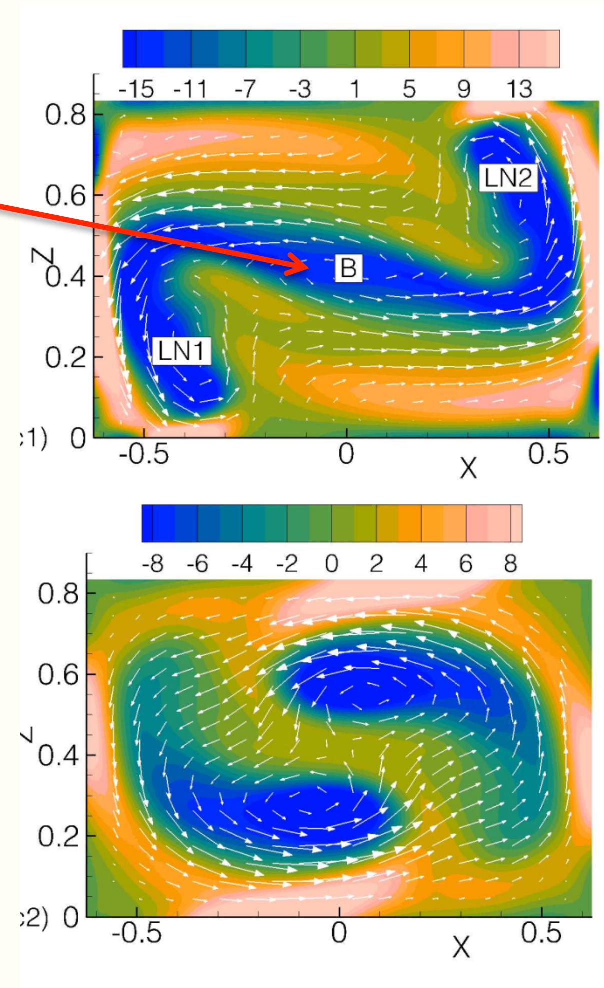
Dynamics in the mixing channel



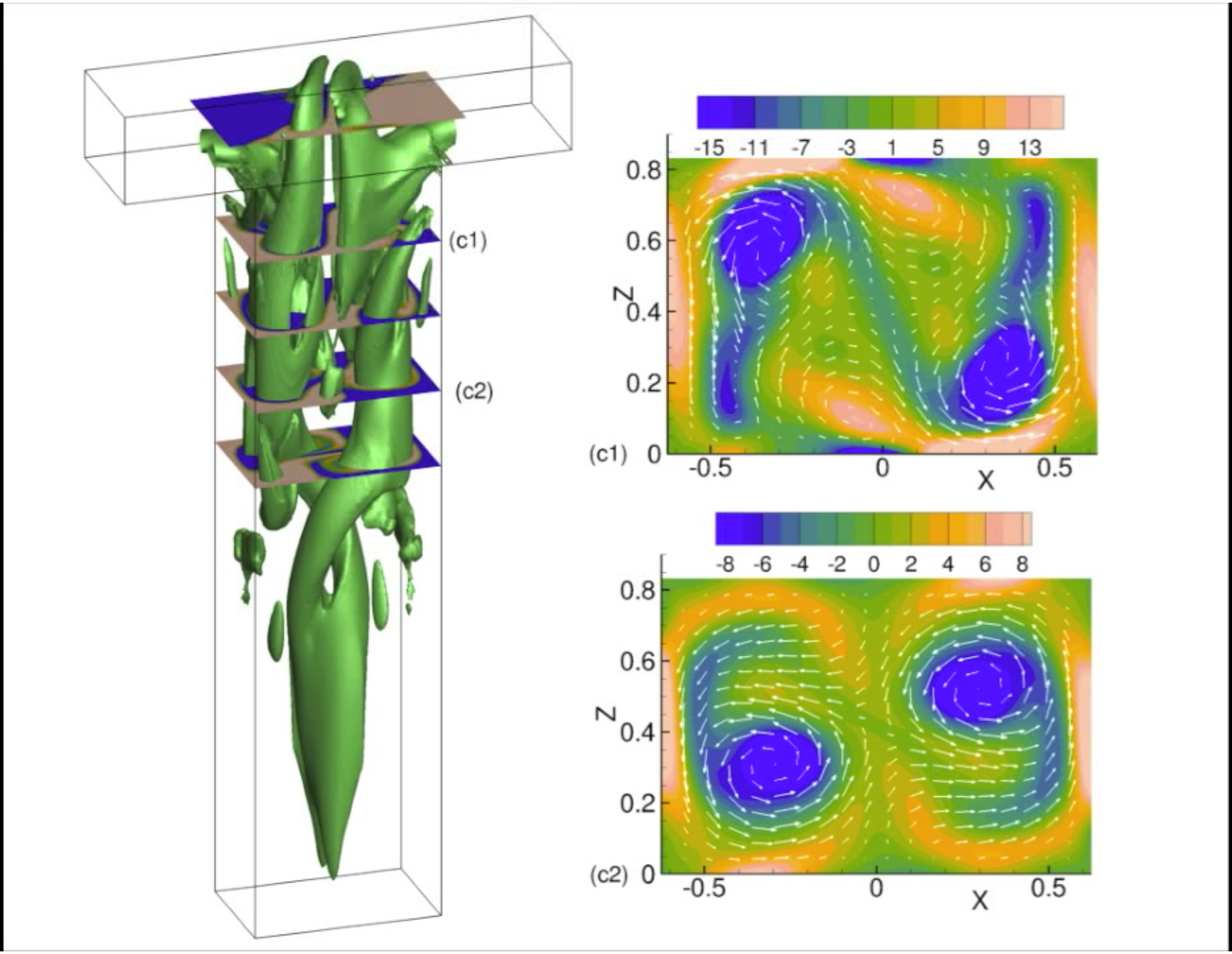
$t=0.74T$

B induces the rotation of the "new" strong legs

the "old" strong legs L1 and L2 are not feed anymore by the separation bubble S at the top of the mixer and they start to disappear



Unsteady asymmetric regime (DNS)

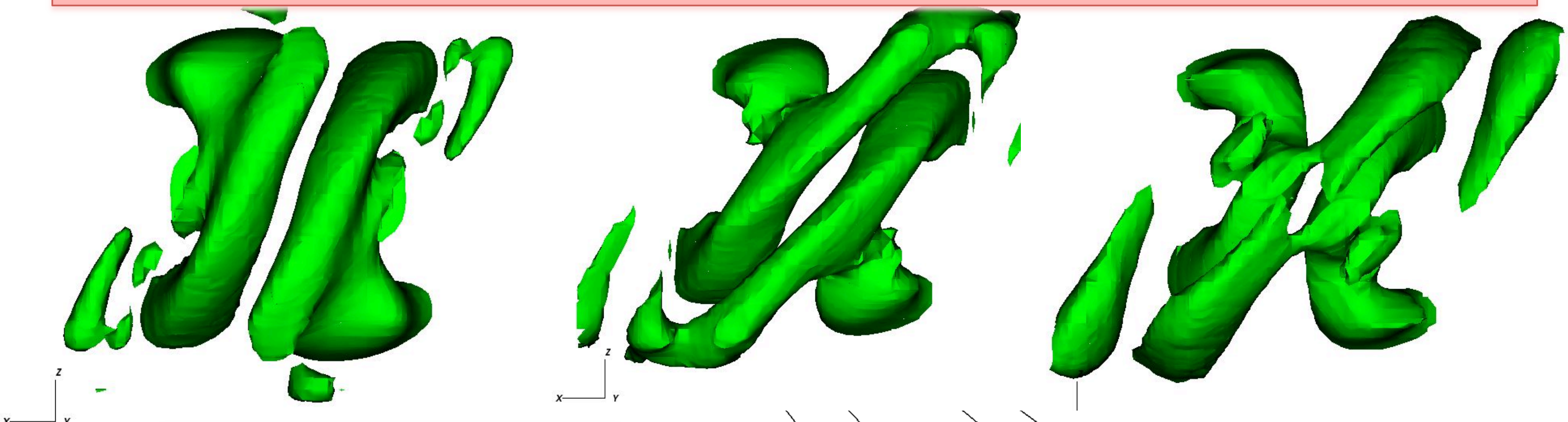


☺ This unsteady regime is also characterized by large asymmetry → high mixing efficiency

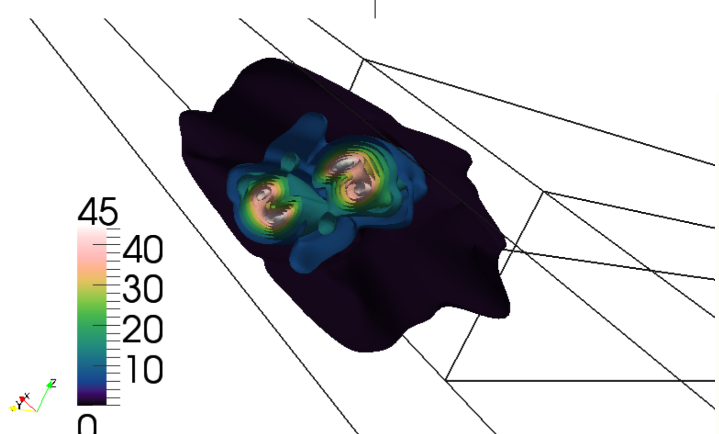
Unsteady asymmetric regime (stability analysis)

- ✓ Stability analysis around the enguflment steady solution (not symmetric fully 3D baseflow) -> fully 3D stability analysis (≈20 milions dofs).
- ✓ Critical Reynolds number $220<Re_c<240$ (in agreement with DNS).
- ✓ One globally unstable eigenmode (not Kelvin-Helmotz instability) is found. It is complex valued and its frequency is in good agreement with the one detected in DNS.

Vortical structures associated with “reconstructed” flow (base flow+unstable mode)

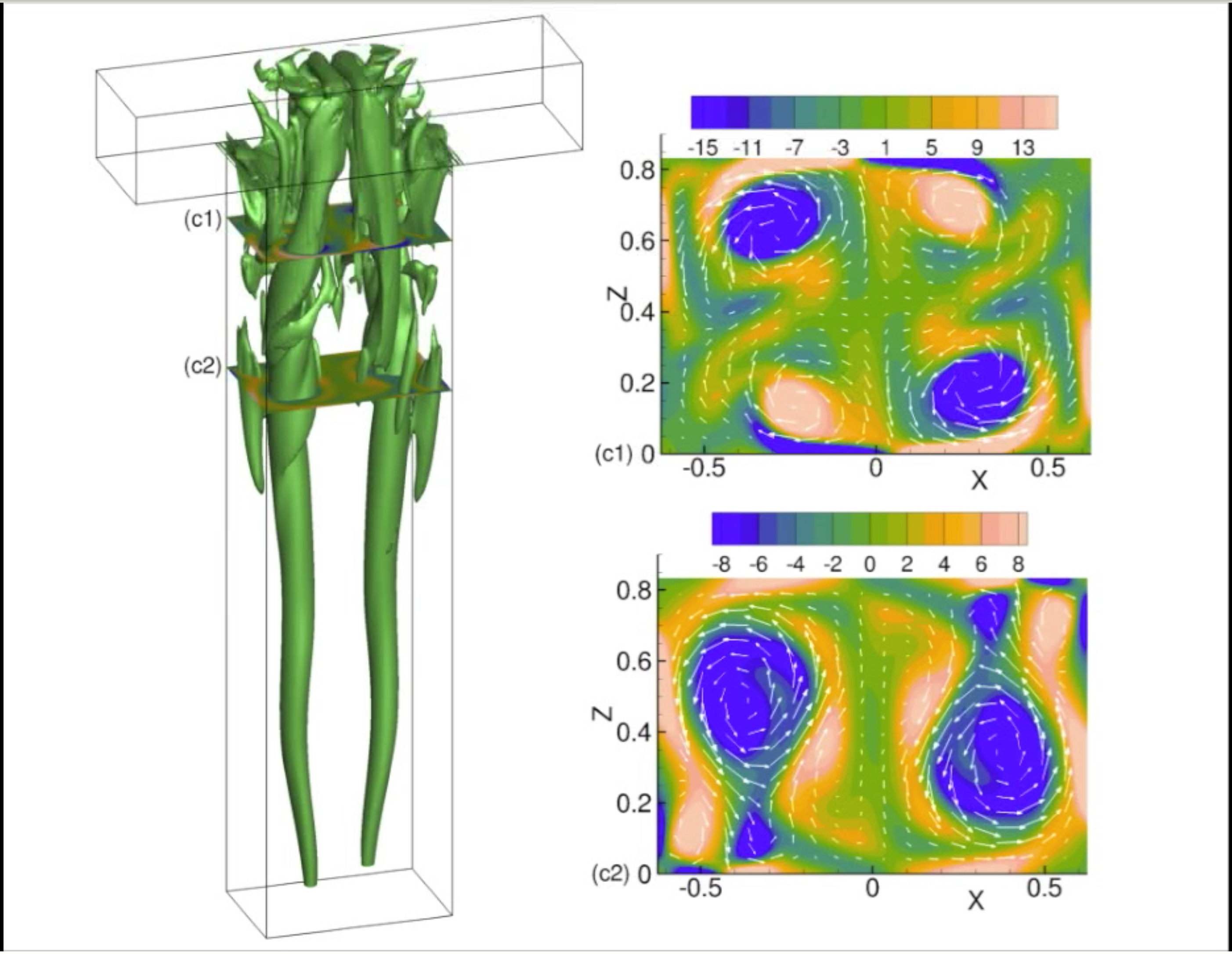


Instability core



Asymmetric as the base flow but again located,inside the recirculation zones where the 3D vortical structures originate.

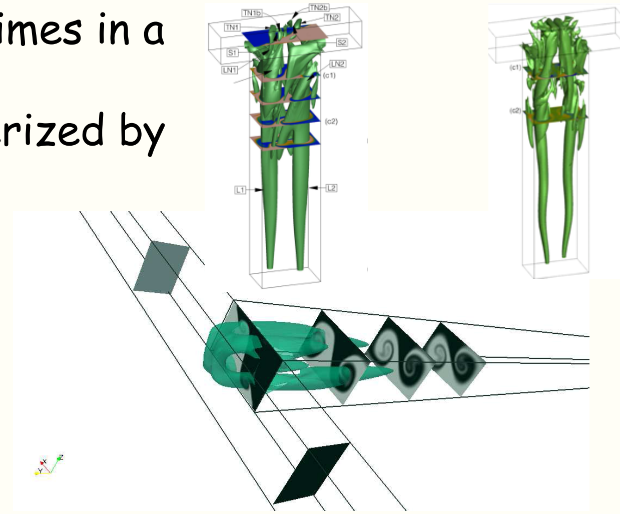
Another regime: unsteady symmetric regime (Re>400)

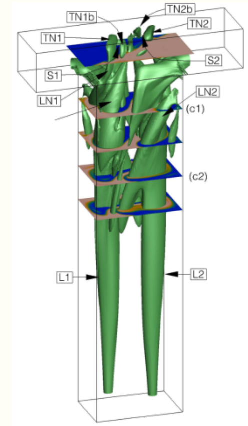


- ✓ The flow periodically switches between an asymmetric configuration (as in engulfment) and a symmetric one similar to the vortex regime.
- ☹ Lower mixing efficiency than in the asymmetric regimes.

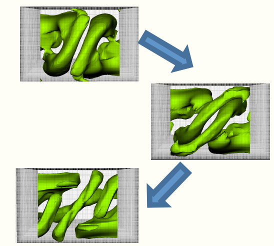
Conclusions

Flow regimes in a T-mixer characterized by DNS

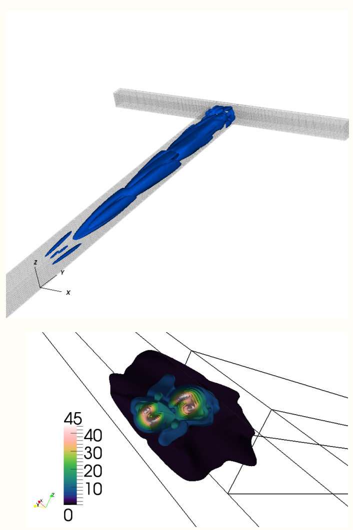


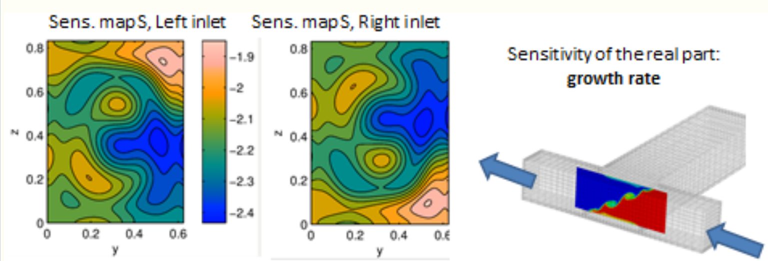


Key role of 3D vortical structures forming at the confluence of the two streams



- 3D global stability analysis
- Localization of the core of the instability (analysis and control)





The sensitivity analysis shows that a not-fully developed inlet velocity profile delays the onset of both steady and unsteady engulfment regimes

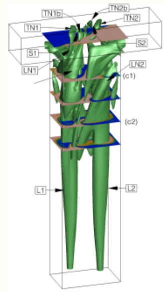
Work in progress/Future work

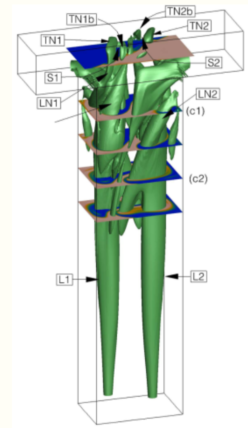
- ✓ Effects of T-mixer geometry (e.g. inlet channel aspect ratio) on the different flow regimes (submitted for publication)
- ✓ Non-Newtonian fluids
- ✓ Two different fluids entering in the T-mixer

Thank you!

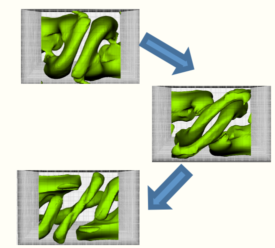
Conclusions

Flow regimes in a T-mixer characterized by DNS





Key role of 3D



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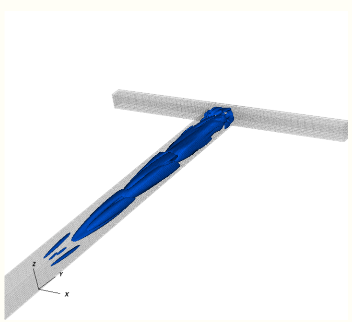

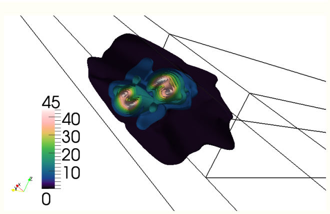

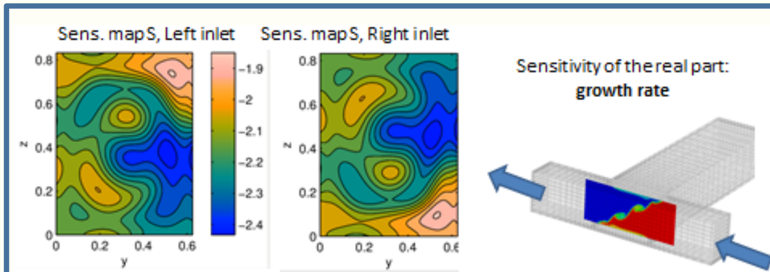

The sensitivity analysis shows that for the non-fully developed case the flow tends to be more stable also for the unsteady engulfment

Sensitivity to perturbation of the inlet velocity conditions

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DNS and experiments in the literature show high sensitivity of the flow regimes
In a T-mixer to pertubrations of the inlet conditions (fully or not-fully developed flow)

This motivates a systematic sensitivity analysis through adjoint methods

We consider a perturbation of this form $\delta \mathbf{U}_i = \delta U_i \mathbf{n}$

Result in the following compact form: $$\delta\sigma = \iint_{in} \delta U_i S(y,z) d\Omega$$

inlet boundaries of the comp. domain

S : **sensitivity map** of the eigenvalue with respect to a localized modification of the wall normal component of the inflow velocity, computed on the inlet surface.

U_i is the perturbation in the direction of the external normal to the comp. domain

Sensitivity to perturbation of the inlet velocity conditions

Marquet et al., jfm 2008

$$\delta\sigma = \iint_{in} \delta U_i S(y,z) d\Omega$$

sensitivity map S

direct stability equations

$$\sigma \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{U}_b + \mathbf{U_b} \cdot \nabla \mathbf{u} + \nabla p - \frac{1}{Re} \nabla^2 \mathbf{u} = \mathbf{0}$$

$$\nabla \cdot \mathbf{u} = 0$$

Adjoint baseflow equations

$$\nabla \mathbf{U_b} \cdot \mathbf{U}_b^{+} - \mathbf{U_b} \cdot \nabla \mathbf{U_b}^{+} + \nabla P_b^{+} - \frac{1}{Re} \nabla^2 \mathbf{U_b}^{+} = \hat{\mathbf{u}}^* \cdot \nabla \hat{\mathbf{u}}^{+} - \nabla \hat{\mathbf{u}}^* \cdot \hat{\mathbf{u}}^{+}$$

$$\nabla \cdot \mathbf{U_b}^{+} = 0$$

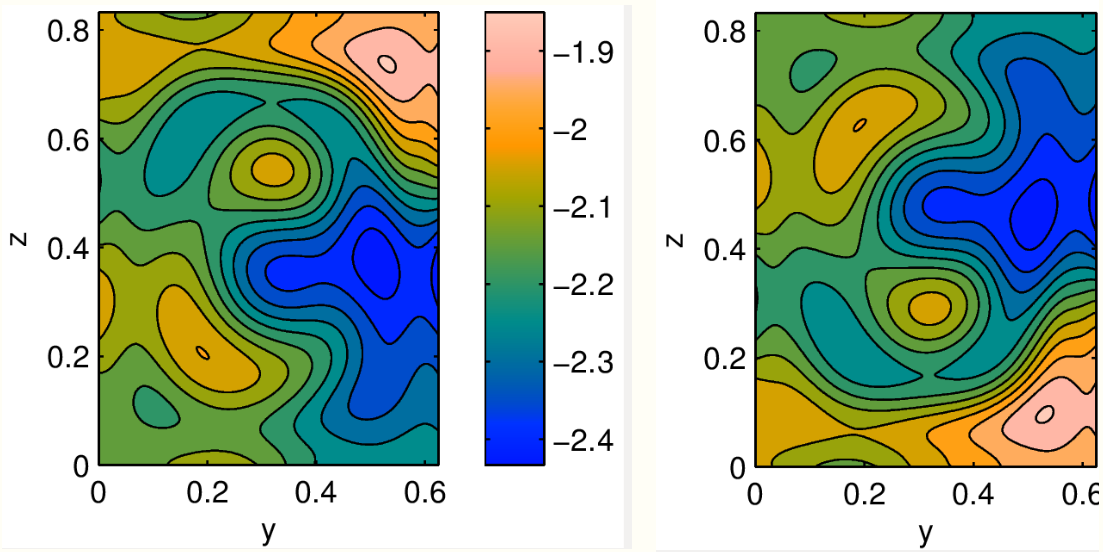
Adjoint stability equations

$$\sigma^* \mathbf{u}^{+} + \nabla \mathbf{U}_b \cdot \mathbf{u}^{+} - \mathbf{U_b} \cdot \nabla \mathbf{u}^{+} + \nabla p^{+} - \frac{1}{Re} \nabla^2 \mathbf{u}^{+} = \mathbf{0}$$

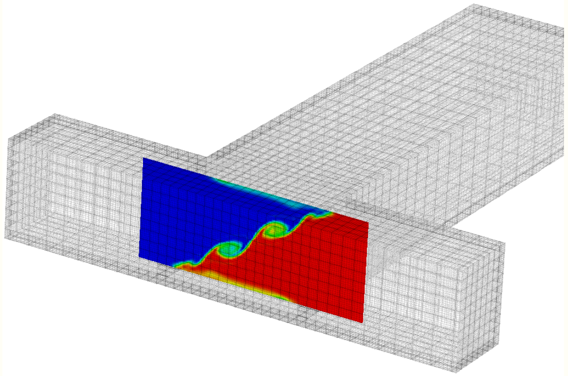
$$\nabla \cdot \mathbf{u}^{+} = 0$$

Sensitivity of growth rate to a perturbation of the inlet velocity conditions

Sens. map S, Left inlet　　Sens. map S, Right inlet



Sensitivity of the real part:
growth rate



- **The two maps are not symmetric due to the asymmetry of** the baseflow
- A constant increase of inflow velocity leads to a de-stabilization (obvious result)
- **An undeveloped generalized Poiseiulle flow is more stable (equal mass rate)**
- An antisymmetric perturbation at the inlet section can have a stabilizing effect

Sensitivity of growth rate to a perturbation of the inlet velocity conditions

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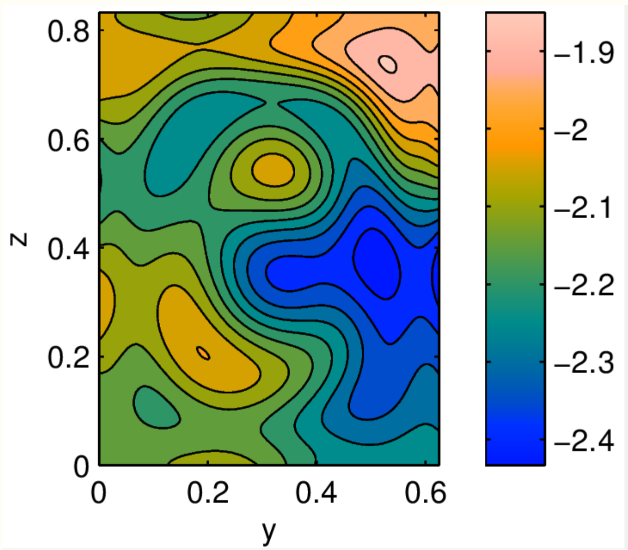
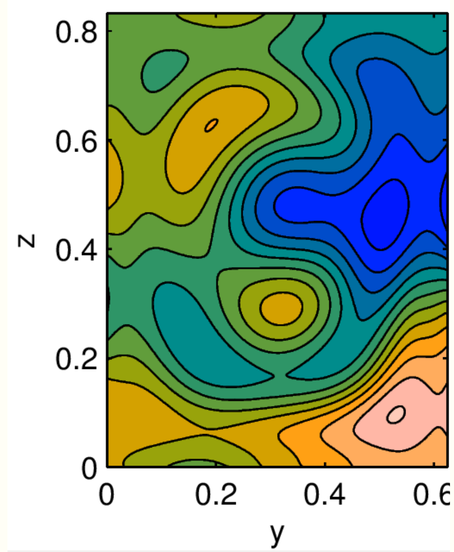
Sens. map S, Left inlet



Sens. map S, Right inlet



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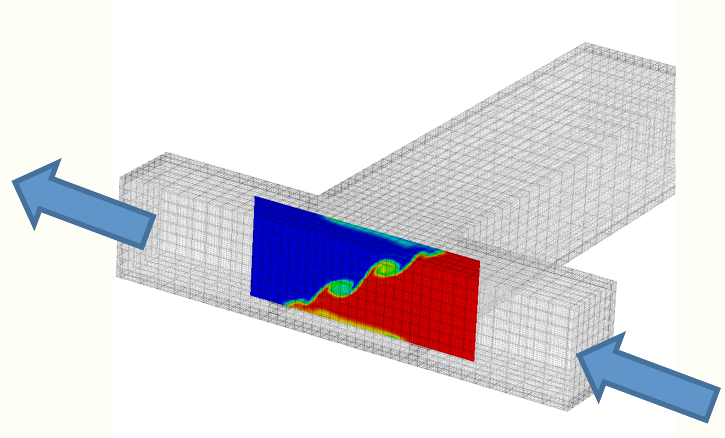

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