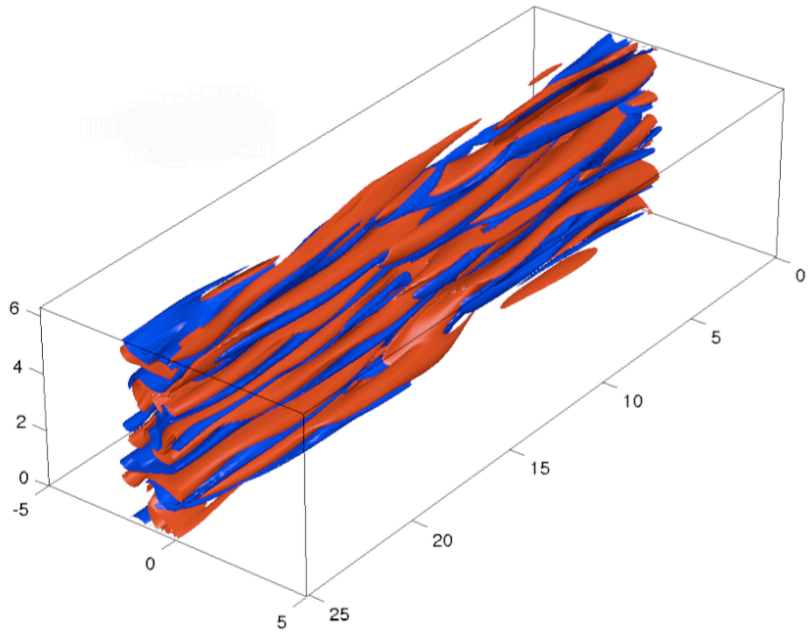
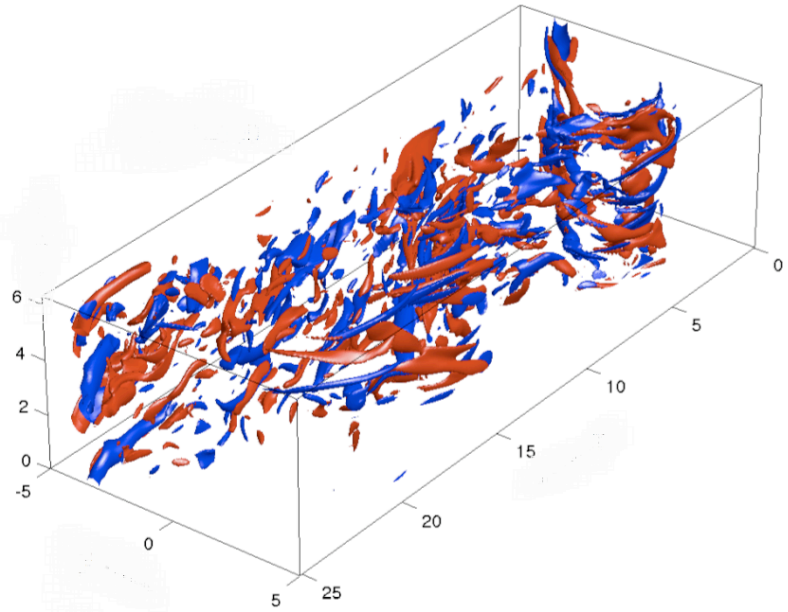
Inertial and Barotropic Instability

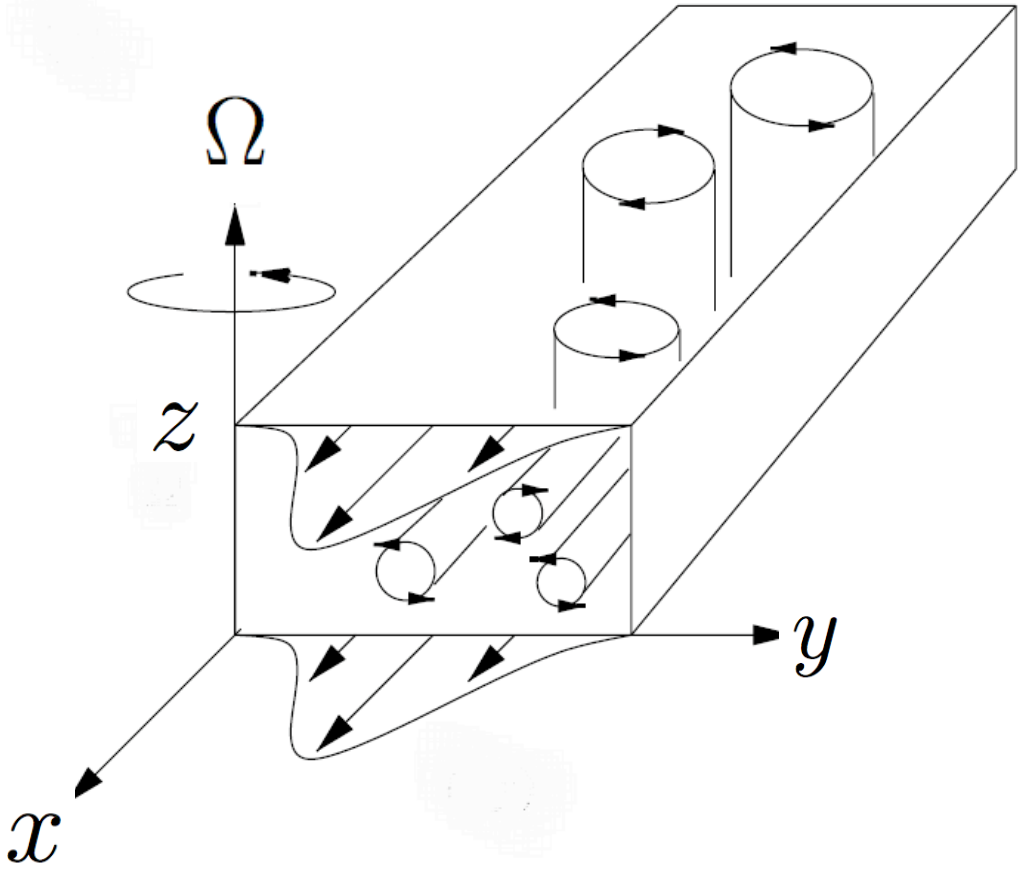




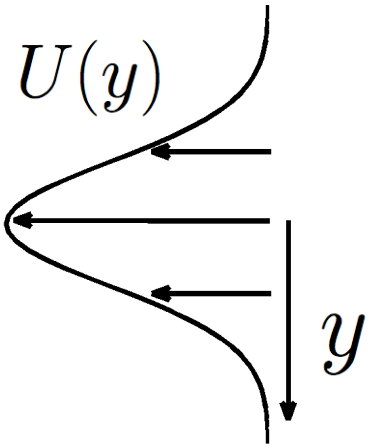
Rudolf Christiaan Kloosterziel

Inertial and Barotropic Instability

HOMOGENEOUS ROTATING FLUID

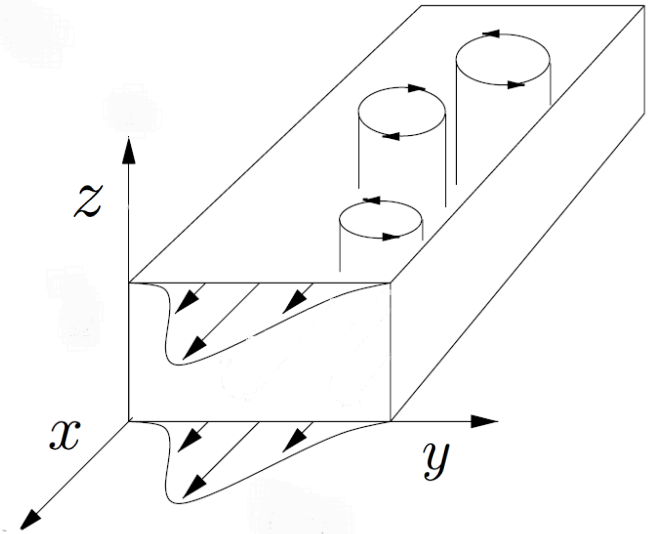


top view



Carnevale, Kloosterziel & Orlandi 2013 *J. Fluid Mech.* **725**, 117–151.

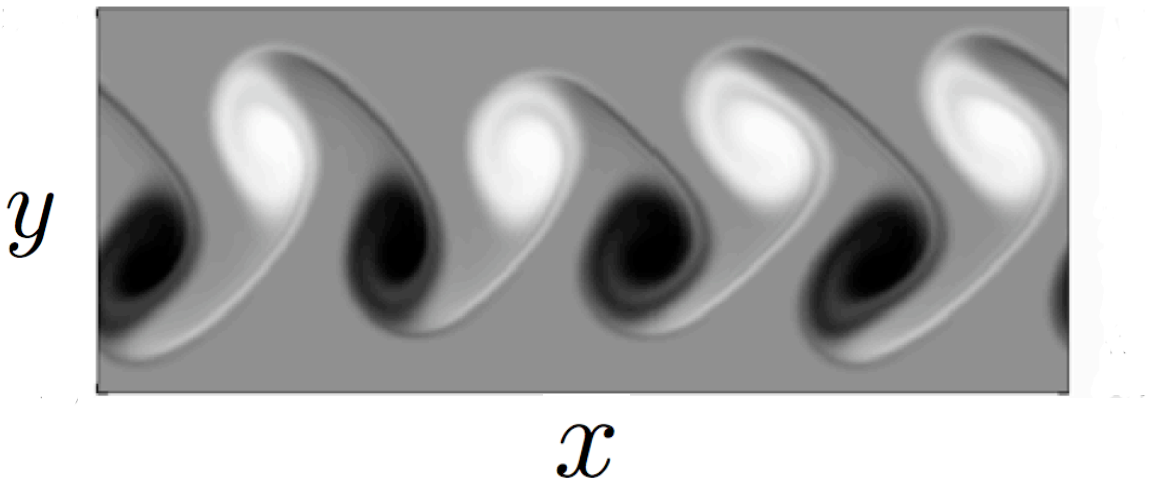
Kloosterziel, Orlandi & Carnevale 2007 *J. Fluid Mech.* **583**, 413–422.



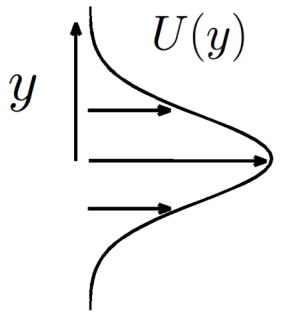
BAROTROPIC INSTABILITY

$$\frac{\partial}{\partial z} = 0$$

vertical vorticity ω_z



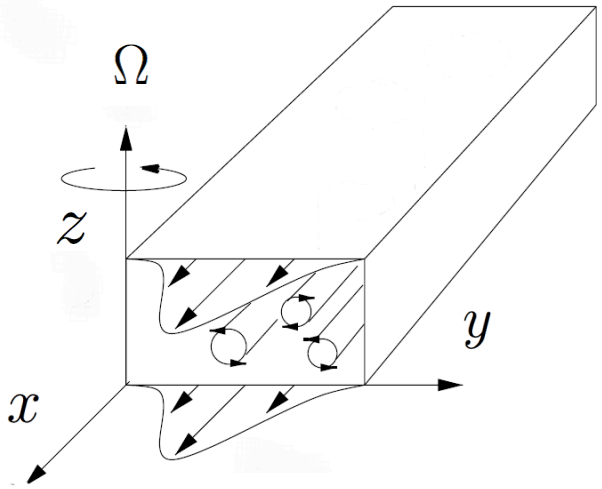
JET



INERTIAL INSTABILITY $\dfrac{\partial}{\partial x} = 0$

Coriolis

$f = 2\Omega$

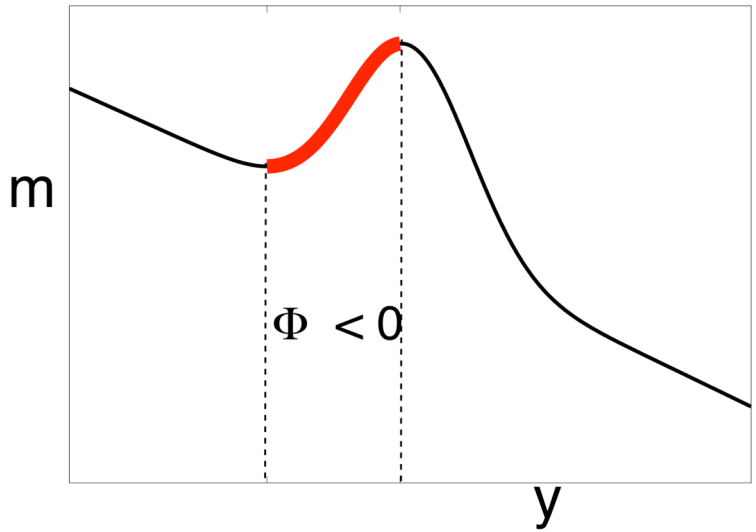


ABSOLUTE MOMENTUM

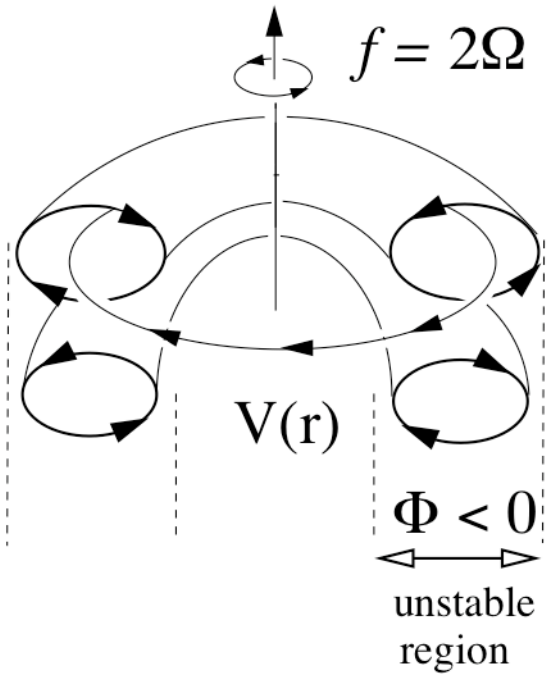
$$m = U - fy$$

UNSTABLE

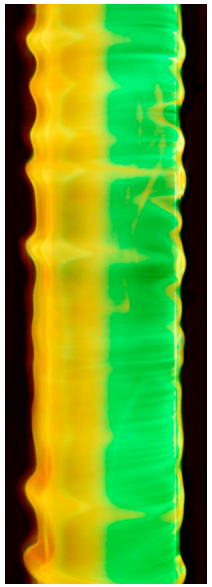
$$\Phi = f\left(f - \frac{\mathrm{d}U}{\mathrm{d}y}\right) < 0 \qquad \frac{\mathrm{d}m}{\mathrm{d}y} > 0$$

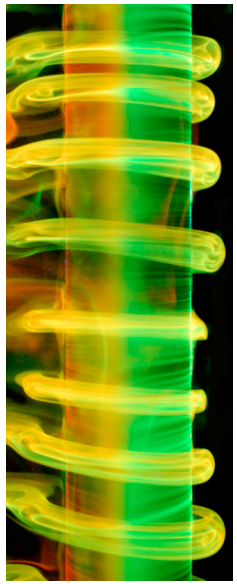


instability



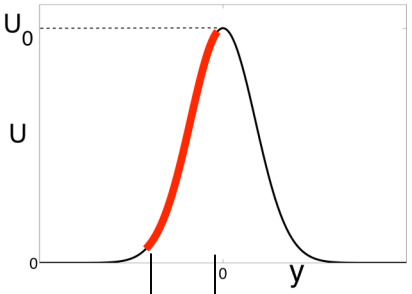
Taylor-Couette Vortices

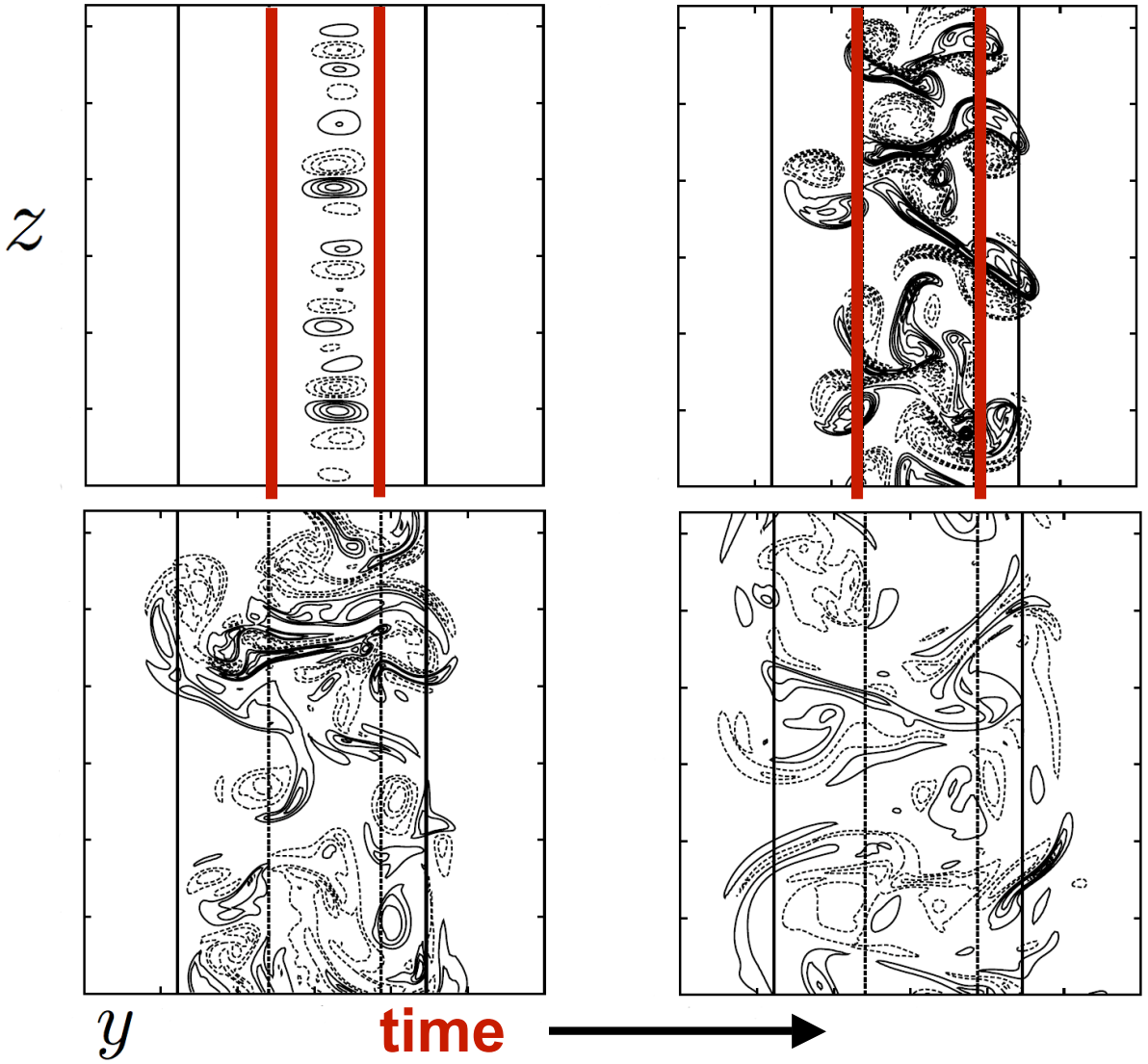




Gaussian Jet $U = U_0 \exp(-y^2/L^2)$

zonal vorticity ω_x



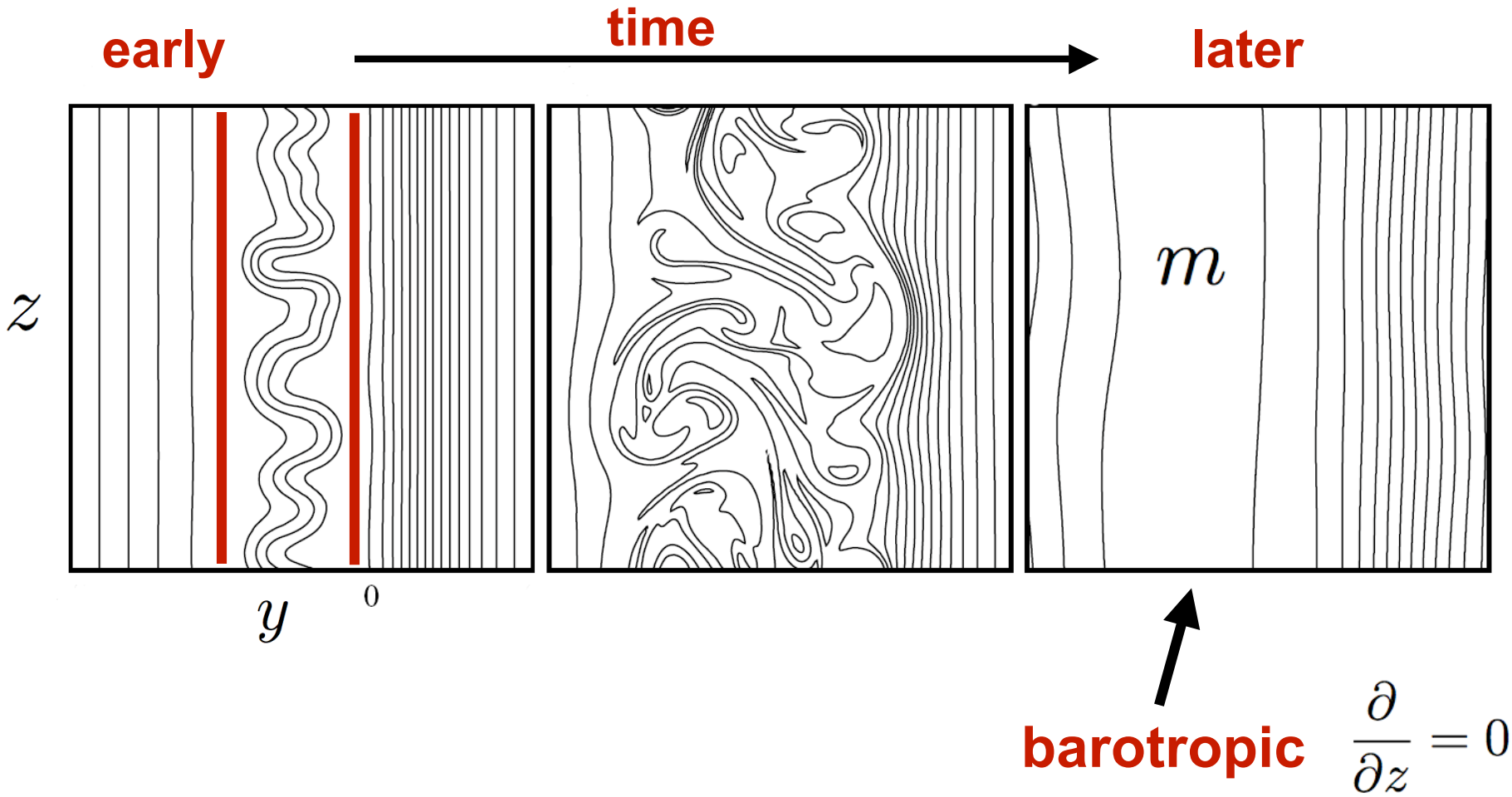


instability

$$\frac{\mathrm{d}m}{\mathrm{d}y} > 0$$

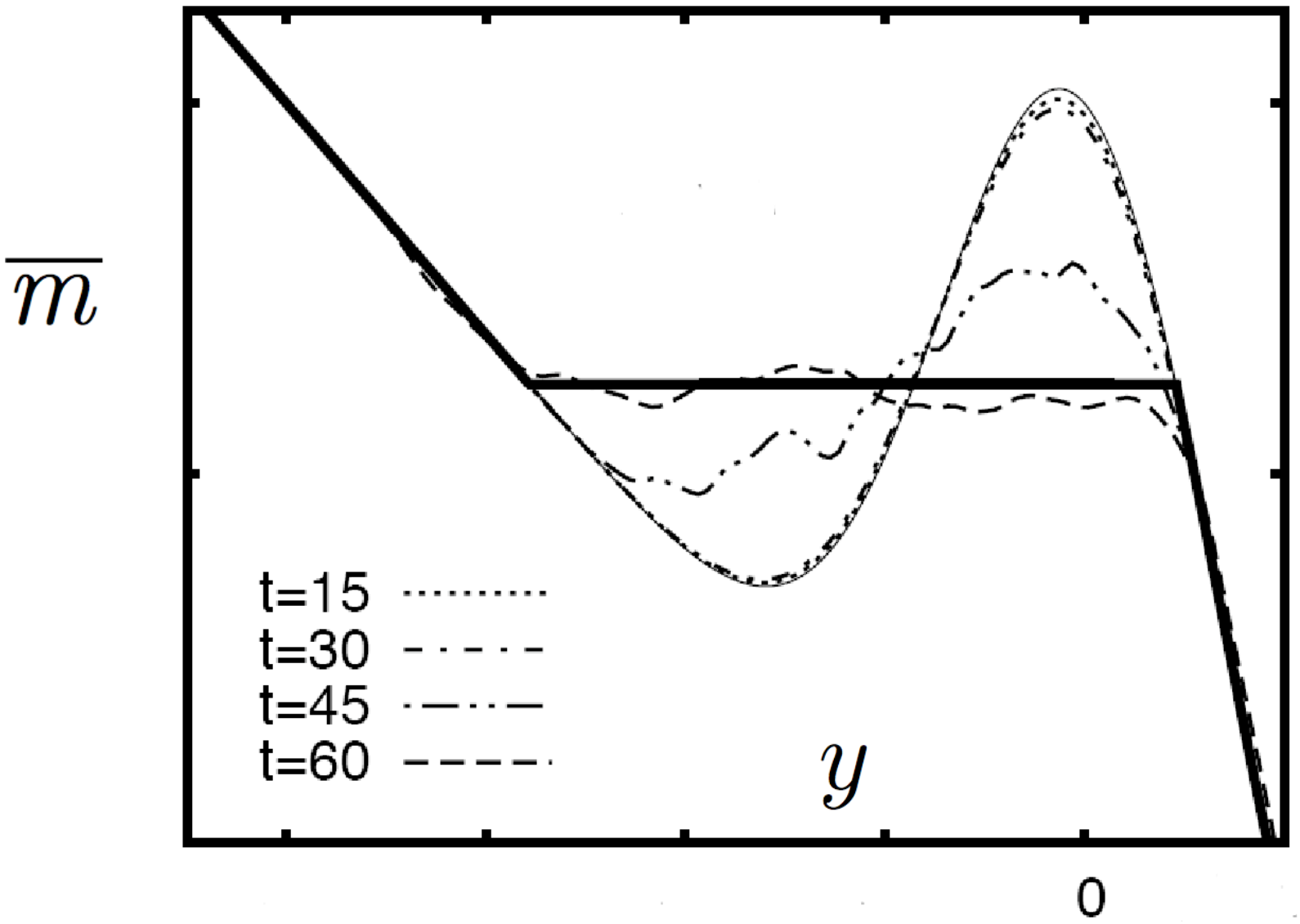
ABSOLUTE MOMENTUM

$$m(y,z,t) = u(y,z,t) - fy$$

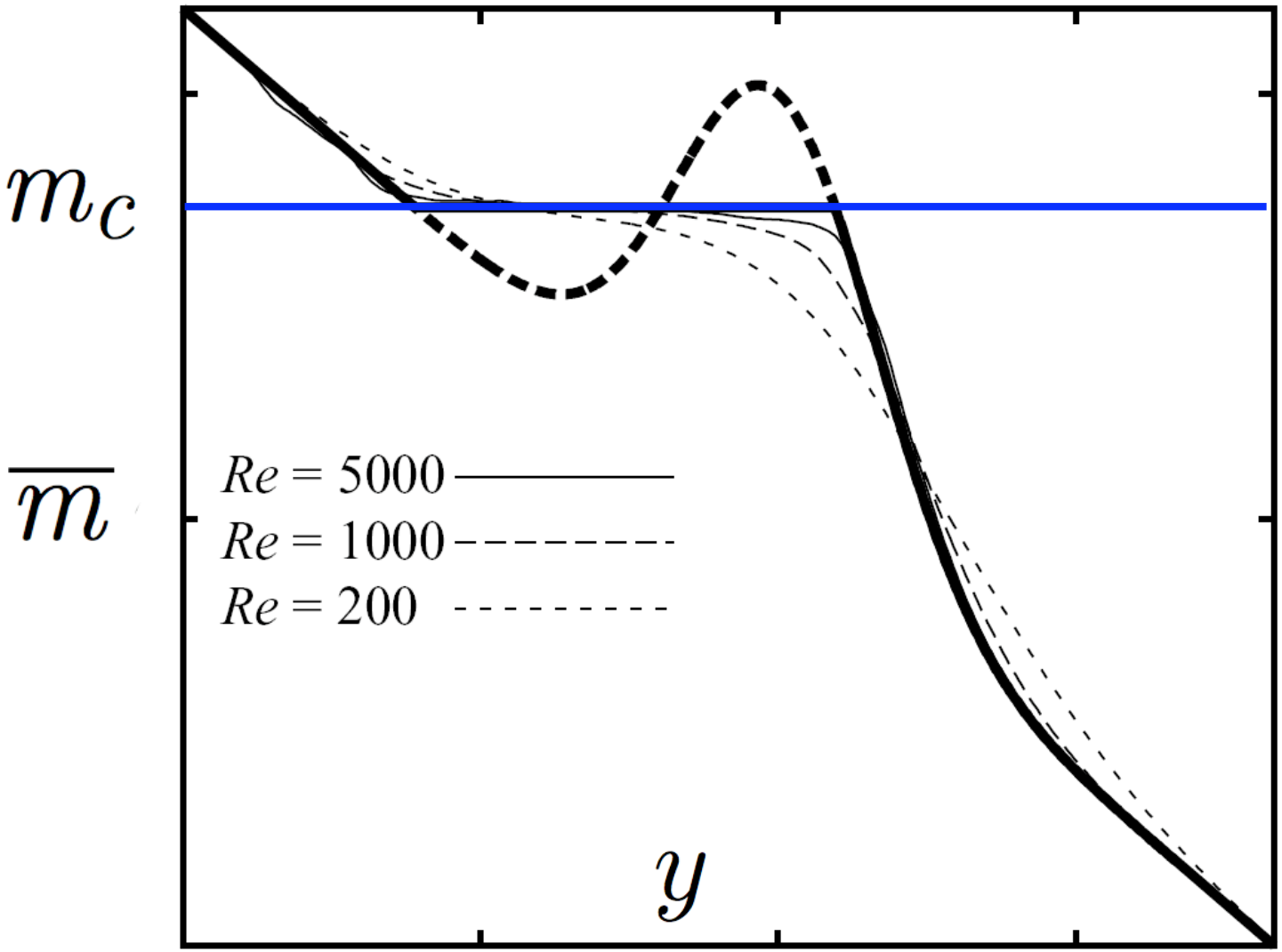


VERTICAL AVERAGE

$$\overline{m}(y,t) = \int m(y,z,t)\,\mathrm{d}z$$



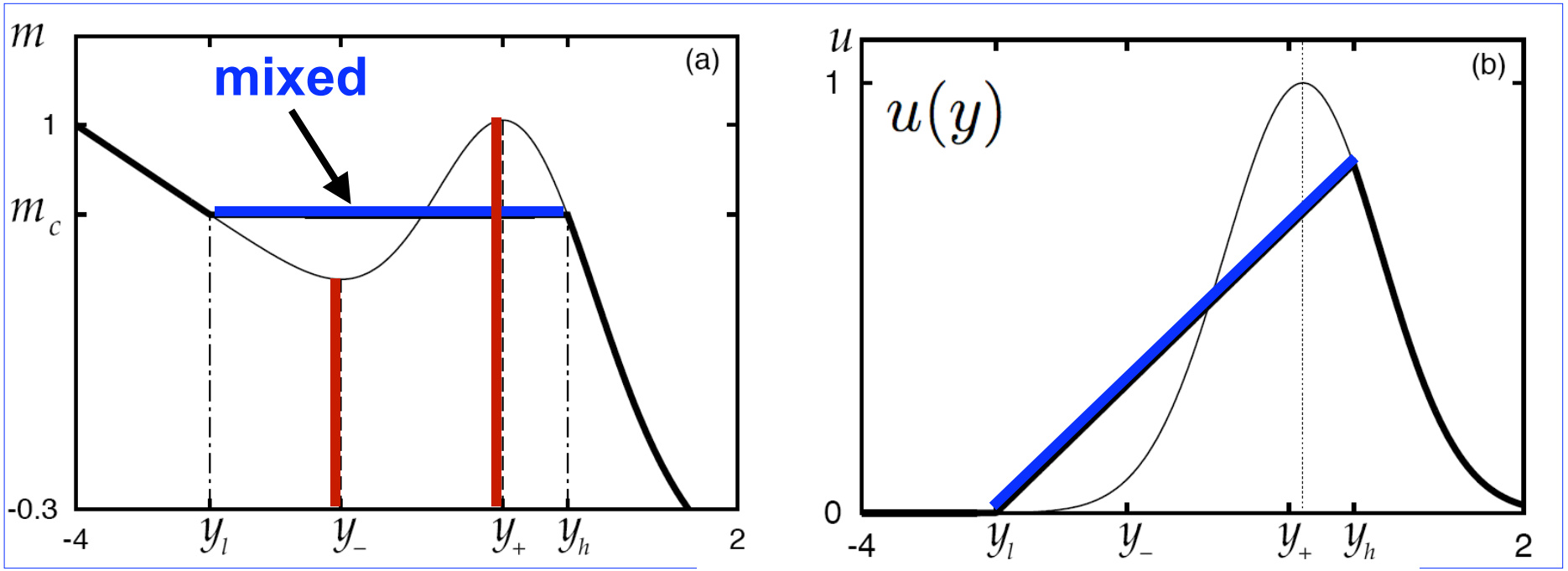
HIGH REYNOLDS NUMBER



INERTIAL INSTABILITY

$$\frac{\mathrm{d}m}{\mathrm{d}y} > 0$$ **unstable** → **mixing** → **stable** $$\frac{\mathrm{d}m}{\mathrm{d}y} \leq 0$$

adjusted stable jet



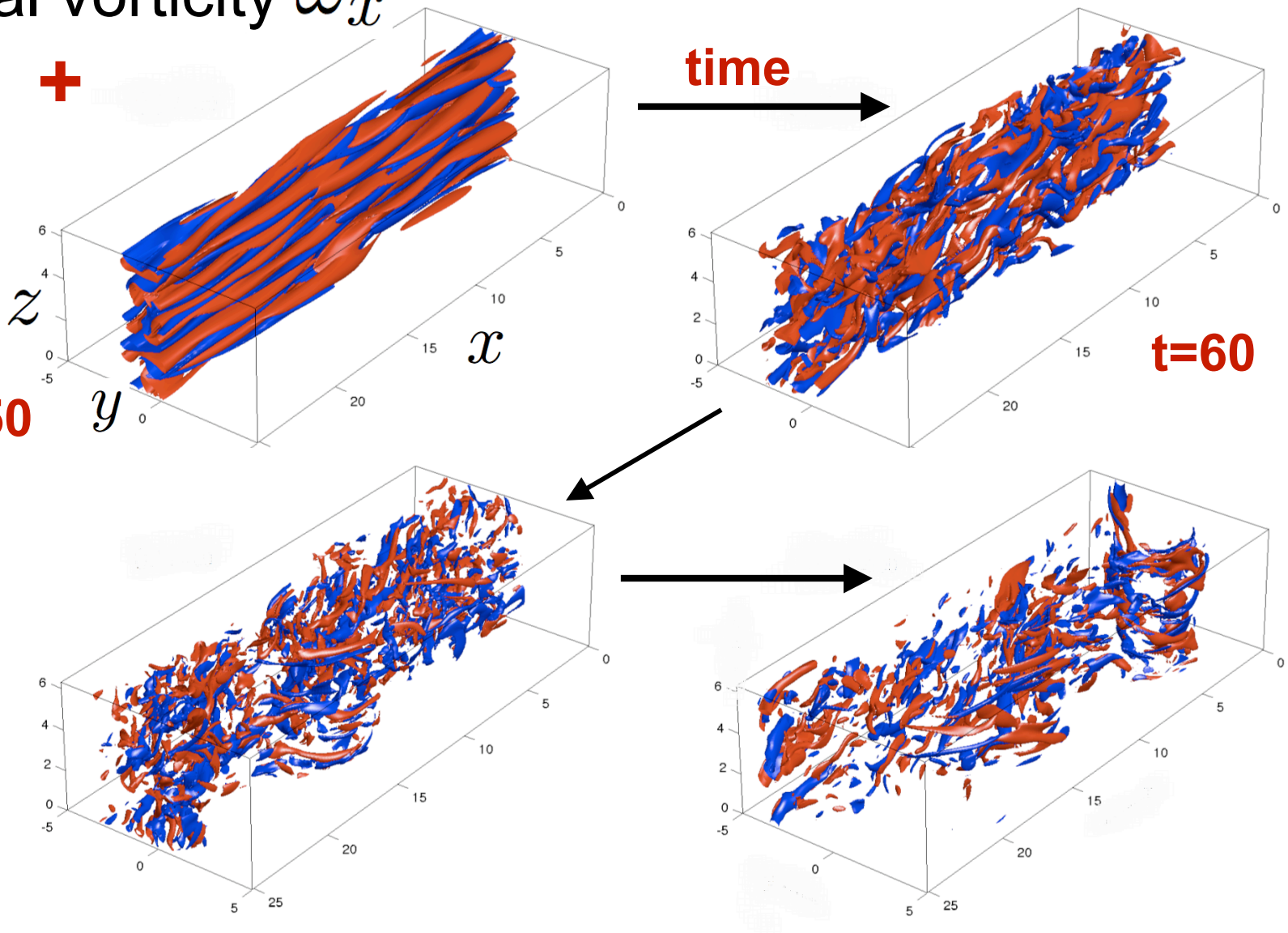
conservation *m*

$$\int_{y_l}^{y_h} (m_c - m(y))\,\mathrm{d}y = 0$$

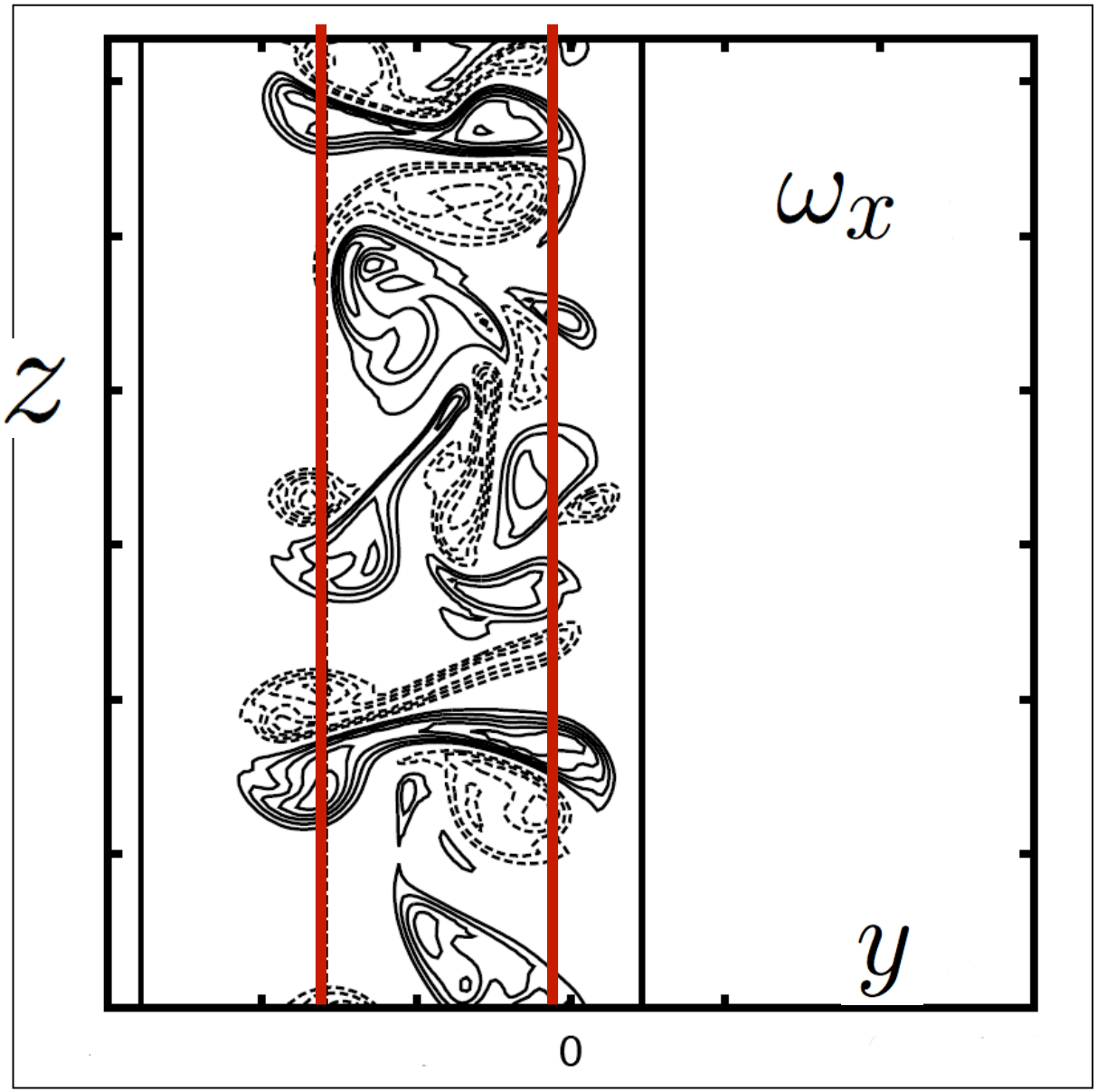
Inertial + Barotropic 3D

zonal vorticity ω_x

— +



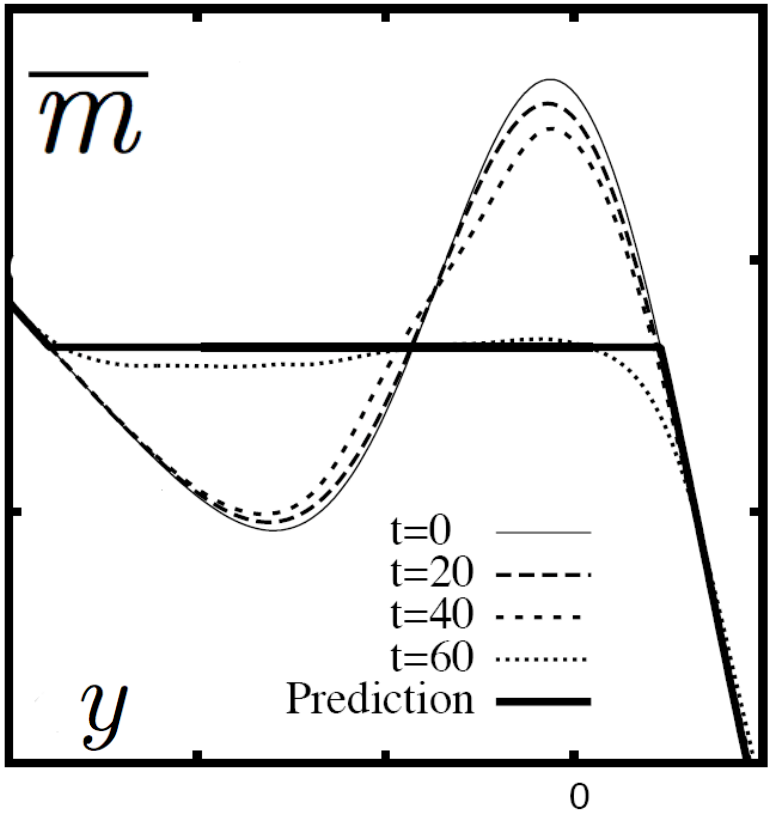
ZONAL VORTICITY t=50

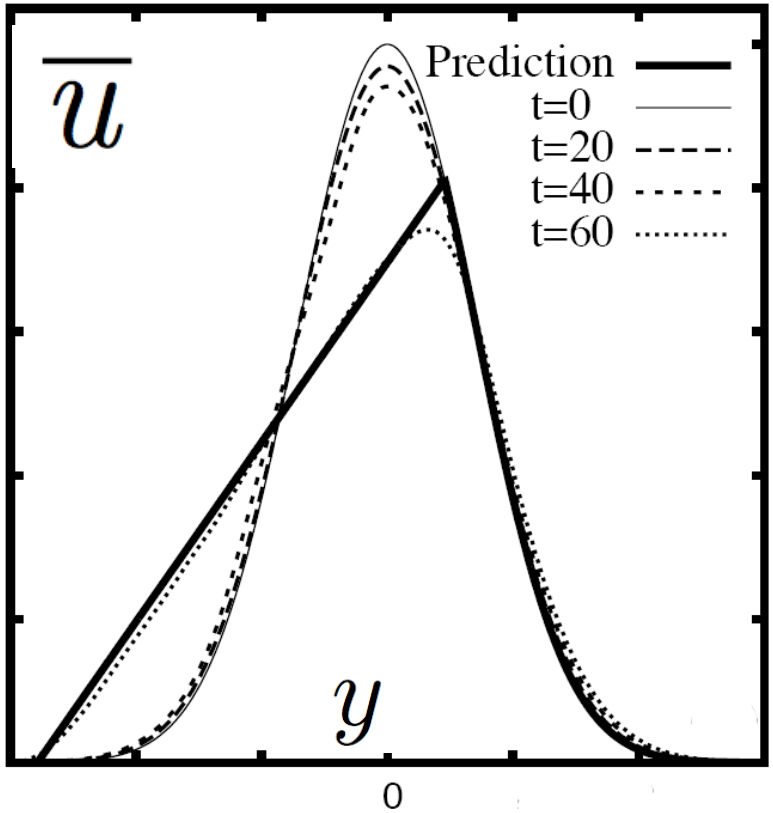


vertical + zonal average

$$\overline{m}(y,t) = \iint m(x,y,z,t)\,\mathrm{d}x\,\mathrm{d}z$$

$$\overline{u}(y,t) = \iint u(x,y,z,t)\,\mathrm{d}x\,\mathrm{d}z$$

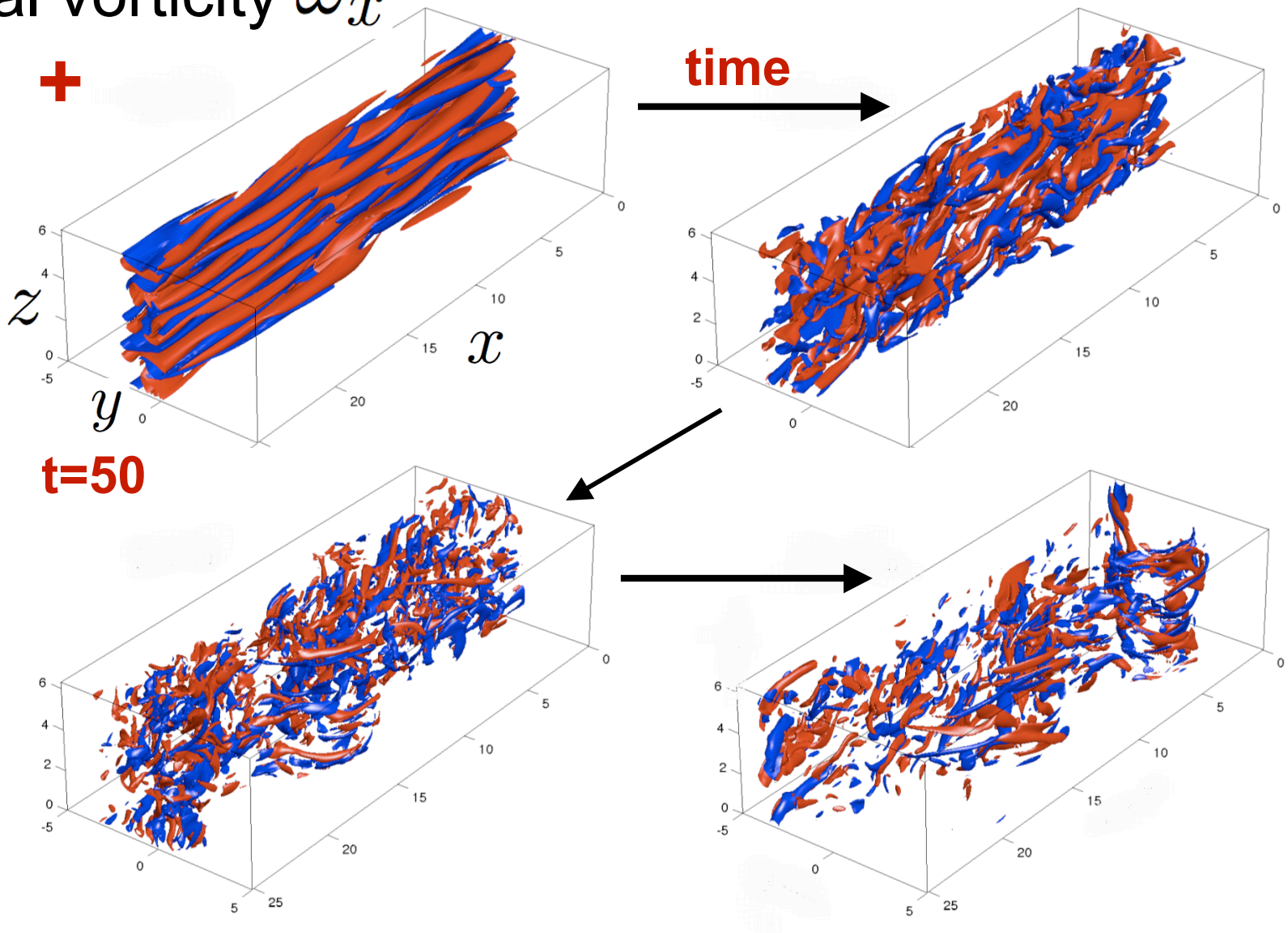




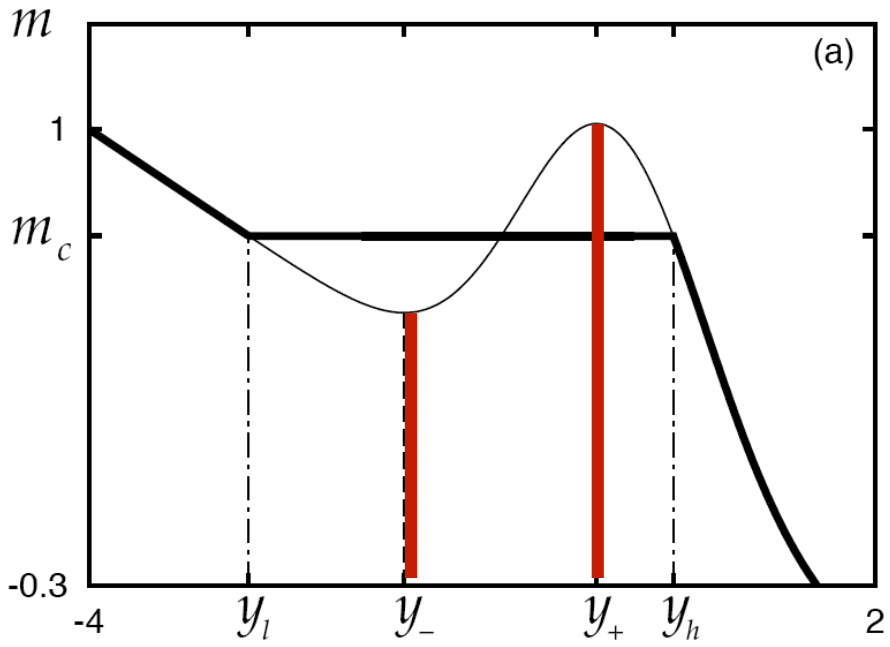
Inertial + Barotropic 3D

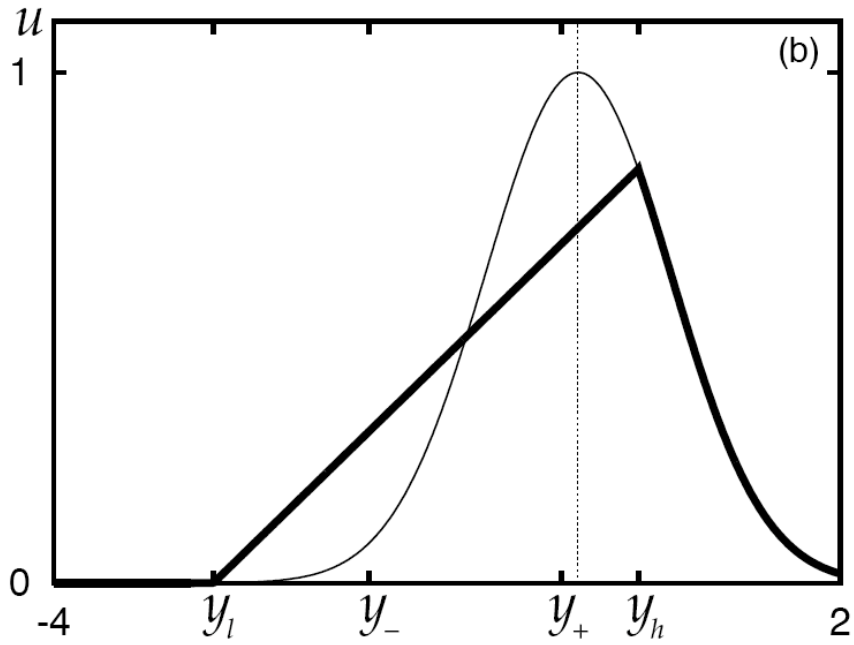
zonal vorticity ω_x

— +



CONCLUSION





$$\int_{y_l}^{y_h} (m_c - m(y))\,\mathrm{d}y = 0 \qquad m = u - fy$$

Absolute momentum mixing in 3D

predicts outcome of nonlinear turbulent dynamics

Carnevale, Kloosterziel & Orlandi 2013 *J. Fluid Mech.*

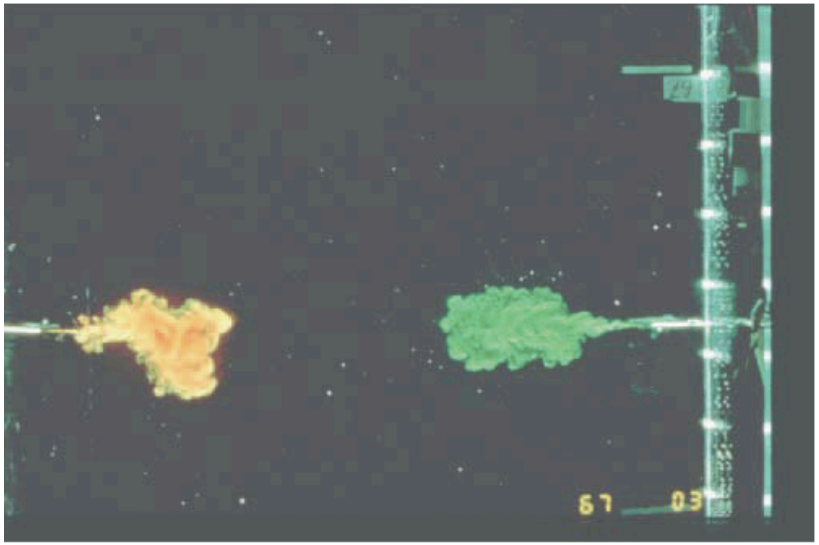


Figure 1(a)

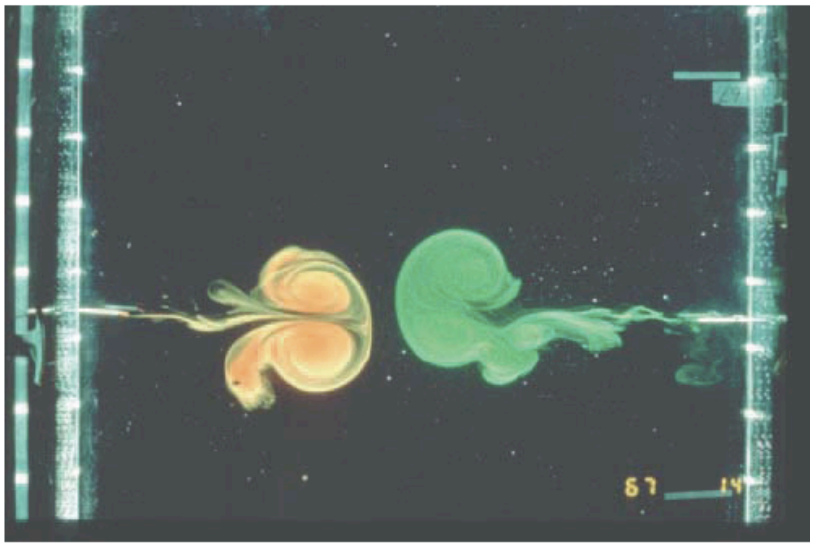
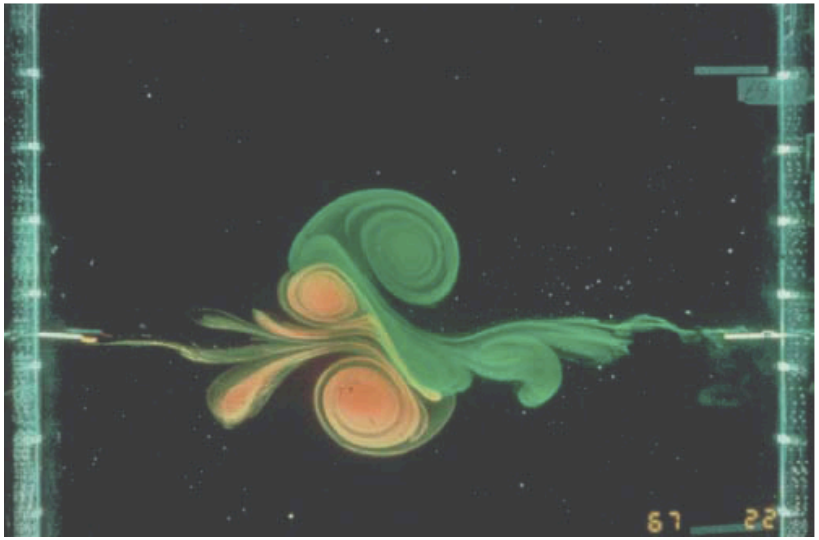
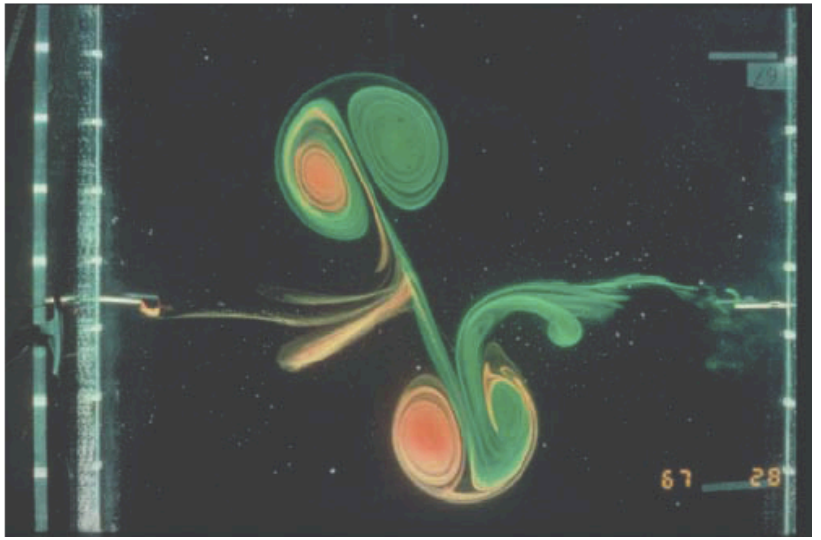
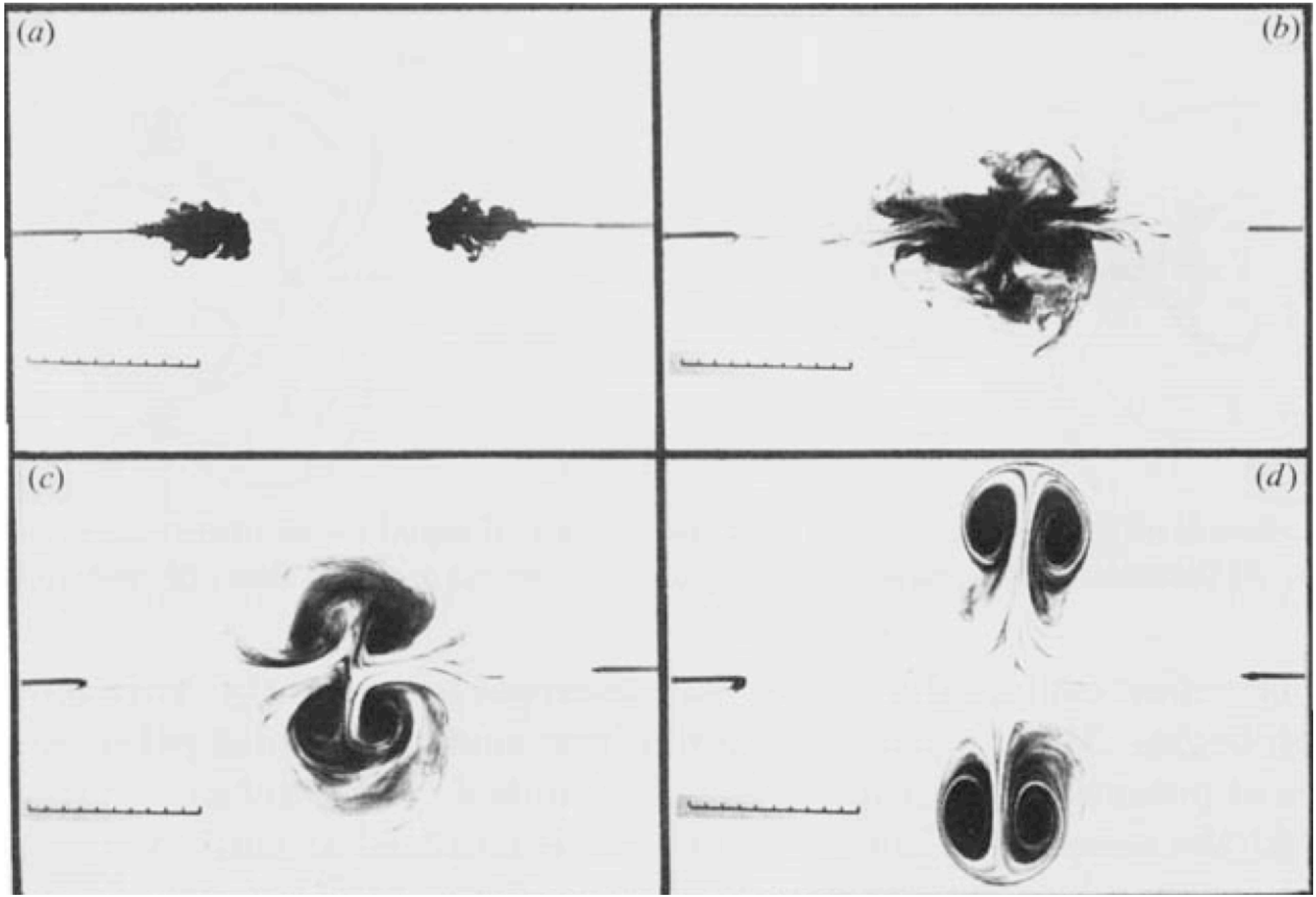


Figure 1b)





van Heijst & Flor Nature 1989



Voropayev & Afanasyev *J. Fluid Mech.* 1992

THANK YOU PAOLO !