Direct numerical simulations of injection flows using cylindrical and spherical coordinates

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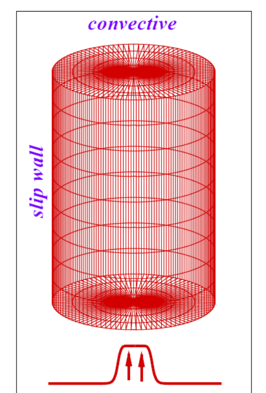
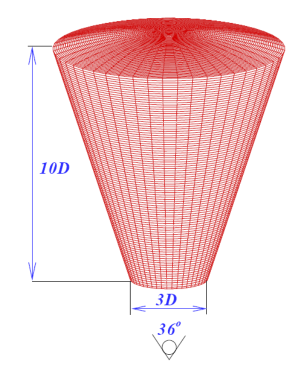
Paolo Orlandi: *A vortical and turbulent life*,
Roma, September 20, 2014.

3D numerical codes: coordinates system

I. Danaila's code (CYL)
University of Rouen
(cylindrical coordinates)



B. J. Boersma's code (SPH)
TU Delft, The Netherlands
(spherical coordinates)



Outline

1. **Navier-Stokes numerical codes using cylindrical and spherical coordinates**
2. **Utility of the DNS of axisymmetric injection flows**
   - Investigation of the physics of vortex rings
   - Improve models for the injection velocity profile
   - Application in automotive industry
   - New nice mathematical developments
3. **From fluids to superfluids**
4. **Conclusion**

Outline

3D numerical codes: equations

- Rai & Moin, JCP, 1991.
- Verzicco & Orlandi, JCP, 1996.
- Orlandi, Kluwer Academic Press, 1999.

Cylindrical coordinates: variables
$(q_\theta = v_\theta, q_r = v_r \cdot r, q_z = v_z, p)$.

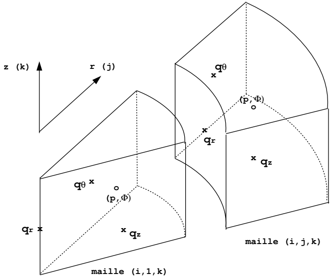
Navier-Stokes equations

- incompressible ($\mathrm{div}\vec{v} = 0$)
- low Mach number approximation ($M \to 0$)

Discretization

Mesh : 3D, staggered

- θ and z directions: uniform grid
- r direction: stretched grid (cyl coordinates).



Spatial discretization: second order finite differences.
Time integration:

- (CYL) convective terms: 3-steps Runge-Kutta method,
- (CYL) diffusive terms: Crank-Nicolson method,
- (SPH) explicit Adams-Bashfort scheme.

Fractional time step (projection) method

for each step of Runge-Kutta:

- momentum equations (ADI factorization)

$$\left(1 - \frac{\alpha_l}{2}\Delta t \mathcal{A}_c\right)\Delta \hat{q}_c^l = \left[\gamma_l \mathcal{H}_c^l + \rho_l \mathcal{H}_c^{l-1} - \alpha_l \mathcal{G}_c p^l + \alpha_l\, \mathcal{A}_c q_c^l\right]$$

- Poisson equation(FFT in θ + cyclic reduction)

$$\mathcal{L}\Phi^{l+1} = \frac{1}{\alpha_l \Delta t}\,\mathcal{D}\vec{\hat{q}}^{\,l}$$

- corrected velocity field

$$q^{l+1} = -\alpha_l \Delta t \mathcal{G}\Phi + \hat{q}^l$$

- scalar equation (TVD scheme)

Navier-Stokes equations for low Mach flows

Idea: remove the pressure waves ($\epsilon = \gamma M^2$)

$$\frac{\partial \rho_0}{\partial t} + \nabla.(\rho_0 \vec{v}_0) = 0$$

$$\frac{\partial \rho_0 \vec{v}_0}{\partial t} + \nabla.\left(\rho_0\ \vec{v}_0 \otimes \vec{v}_0\right) = -\nabla p_1 + \frac{1}{Re}\nabla.\vec{\vec{\tau_0}}$$

$$\frac{\partial \rho_0 Y_0}{\partial t} + \nabla.\left(\rho_0 \vec{v}_0 Y_0\right) = \frac{1}{ReSc}\nabla.\left(\mu \nabla Y_0\right)$$

- New equation ($\rho_0 = 1/T_0$):

$$\frac{\partial \rho_0}{\partial t} = -\vec{v}_0\ \nabla \rho_0 - \frac{1}{T_0}\left[\frac{1}{RePr}\nabla.(\mu\ \nabla T_0)\right]$$

Numerical algorithms for Low-Mach

Different methods, e.g. Adams-Bashforth explicit scheme

- start by integrating ρ equation:

$$\frac{\rho^{n+1}-\rho^n}{\Delta t} = \left[\frac{3}{2}F^n - \frac{1}{2}F^{n-1}\right], \quad \left(\frac{\partial\rho}{\partial t}\right)^{n+1} = \frac{3\rho^{n+1} - 4\rho^n + \rho^{n-1}}{2\Delta t}$$

- prediction step

$$\frac{\rho^{n+1}\hat{q}_c - \rho^n q_c^n}{\Delta t} = \left[\frac{3}{2}G_c^n - \frac{1}{2}G_c^{n-1} - \mathcal{G}_c p^n\right]$$

- correction step

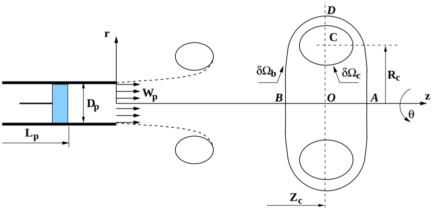
$$\mathcal{L}\Phi = \frac{1}{\Delta t}\left[\mathcal{D}\left(\rho^{n+1}\hat{q}_c\right) + \left(\frac{\partial\rho}{\partial t}\right)^{n+1}\right],$$

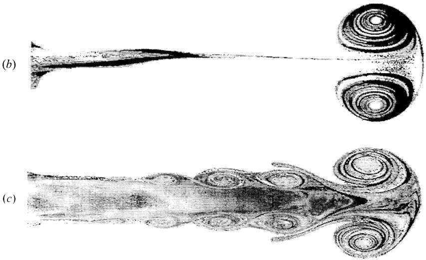
$$(\rho q_c)^{n+1} - \rho^{n+1}\hat{q}_c = -\Delta t \mathcal{G}_c \Phi$$

Outline

The vortex ring as a fundamental flow

- injection of fluid in a quiescent ambiance.

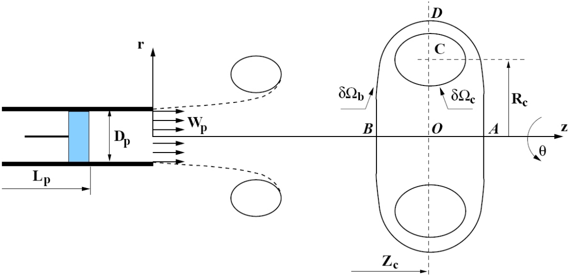


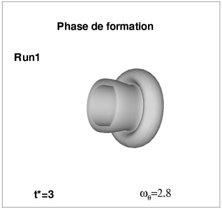


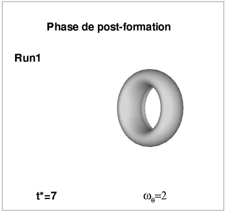
Renewal of fundamental studies:
Gharib et al., JFM, 1998; Kaplanski et al., Phys. Fluids, 2005.

Vortex ring simulations

- idea: simulate separately the pipe-flow and the vortex ring





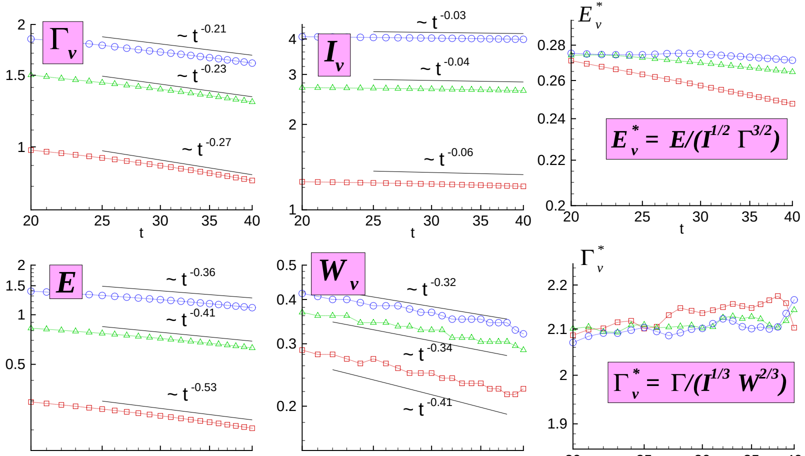


Postformation evolution: evolution laws

I. Danaila and J. Hélie, Physics of Fluids, 2008.

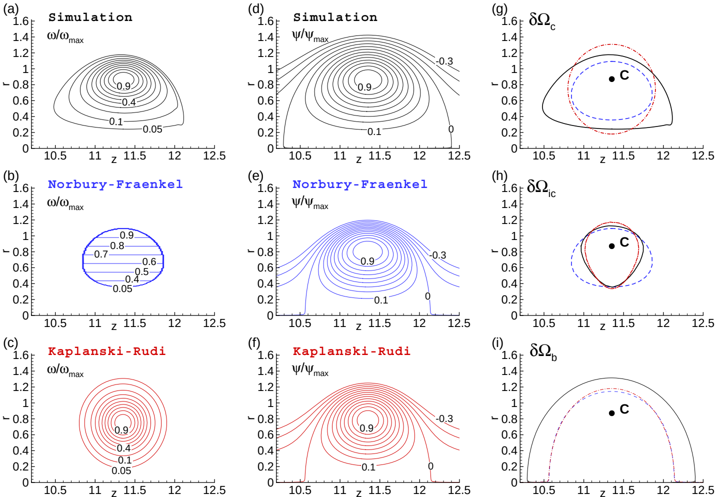
high resolution → good agreement with experiments

reconciliate Dabiri et al., JFM, 2004 and Maxworthy, JFM, 1972.



Postformation evolution: fit to ideal vortex models

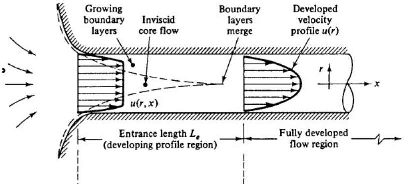
I. Danaila and J. Hélie, Physics of Fluids, 2008.



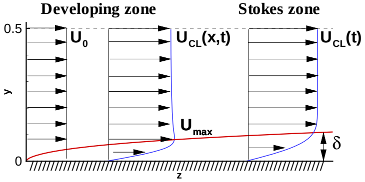
Accurate model for the inflow boundary condition

I. Danaila, C. Vadean and S. Danaila,
Theor. Comput. Fluid Dynamics, 2009.

- Entrance zone + Fully developed zone



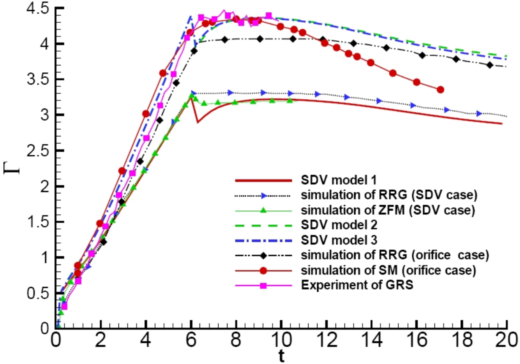
- Inlet zone + Stokes zone



New inflow velocity model: DNS vs experiments

$$U_{\text{SDV}}(t,r) = U_{\text{CL}}(t)\, F_{inj}(t)\, U_b(r,t),$$

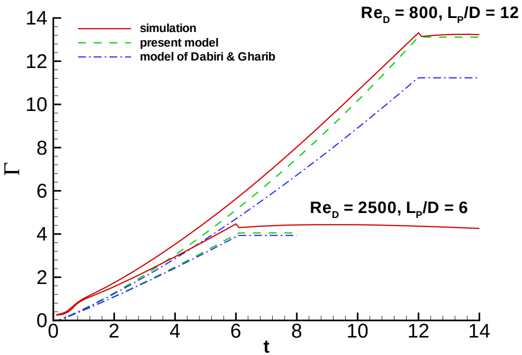
$$U_b(r,t) = \frac{1}{2}\left\{1 + \tanh\left[\frac{1}{4\Theta(t)}\left(1 - \frac{r}{R_{jet}(t)}\right)\right]\right\}$$



LMRS

New inflow velocity model: slug-flow models $\Gamma(t)$

$$\Gamma(t) = \frac{U_0^2 Re_D}{32\beta^2}\left[\frac{B(t)(B(t)-\alpha)}{(B(t)-\alpha)^2+\beta^2} + \frac{\alpha}{\beta}\arctan\left(\frac{B(t)\beta}{\frac{1}{2}-\alpha B(t)}\right)\right]$$



Impulsively starting flows

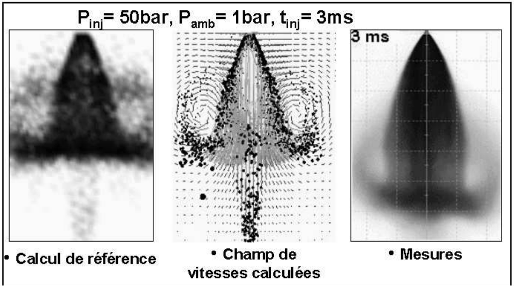
Bio-mechanics, synthetic jet actuators, etc.
Injection flow in internal combustion engines

• Diesel injectors. • New type of gasoline injectors : low pressure, with swirl, multi-point, piezo actuated.

(courtesy of Continental Automotive France)

Vortex rings in internal combustion engines: direct Diesel injection

- Jet collapse: as a result of the opposite-sign vortex interactions (dipoles).
- industrial CFD two-phase flow simulation/ experiment (courtesy Institut Français du Pétrole)



Vortex rings in internal combustion engines: direct Diesel injection

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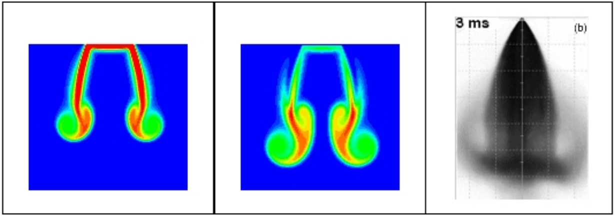
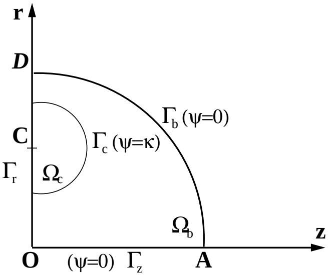


VR problem: mathematical formulation



$$\mathcal{L}\psi = \frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial\psi}{\partial z}\right) + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right) = \begin{cases} -r\omega_0 f(\psi), \text{ in } \Omega_c \\ 0, \text{ in } \Pi\backslash\bar{\Omega}_c, \end{cases}$$

ψ and $\nabla\psi$ are continuous across $\partial\Omega_c$

$$\psi = k \quad \text{on} \quad \partial\Omega_c, \qquad \psi = 0 \quad \text{on} \quad Oz,$$

$$\psi + \frac{1}{2}Wr^2 \to 0 \quad \text{when} \quad r^2 + z^2 \to \infty.$$

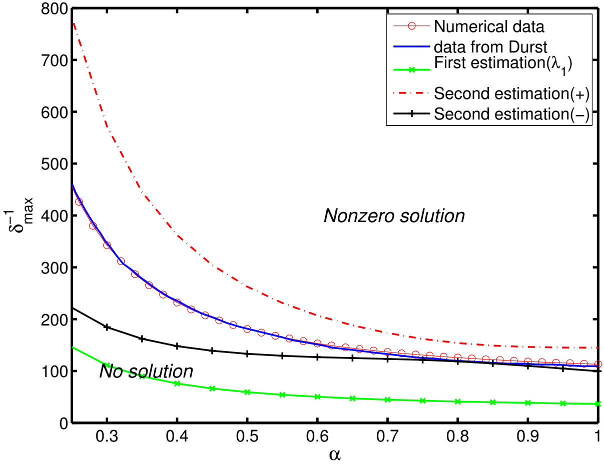
parameters: $W, k, \omega_0, f(r,\psi)$.

Fundamental studies (70' and 80')

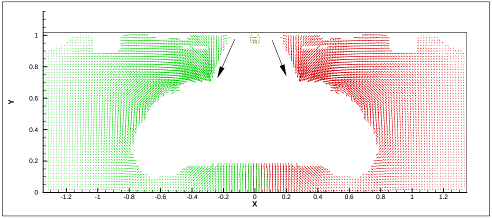
- J. Norbury, Proc. Camb. Phil. Soc., 1972.
- L. E. Fraenkel & M. S. Berger, Acta Math, 1974.
- H. Berestycki, E. F. Cara & R. Glowinski, RAIRO, 1984.
- C. J. Amick & L. E. Fraenkel, Arch. for Rational Mech. and Analysis, 1987.

VR with fixed elliptic $\partial\Omega_b$: non-trivial solutions

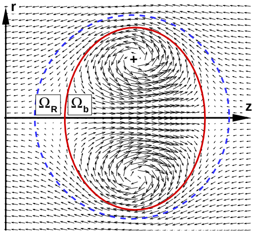
Y. Zhang and I. Danaila, Applied Math. Modelling, 2013.



Reconstruction of the velocity field



PIV image (Siemens Automotive)

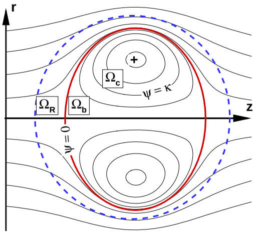


Difficulties

- the solution is not unique,
- the solution depends on the vortex ring model,
- needs careful set of the matching functional,
- numerics based on non-linear fit procedures.

The optimal control problem

Y. Zhang and I. Danaila, J. of Numerical Mathematics, 2012.



$$\operatorname*{Min}_{X \in \mathbb{R}^n} J(\psi) = \int_{\partial\Omega_R} \left| \frac{1}{r} \left(\frac{\partial \psi}{\partial \vec{n}} - \frac{\partial \psi_{exp}}{\partial \vec{n}} \right) \right|^2 dS$$

subject to

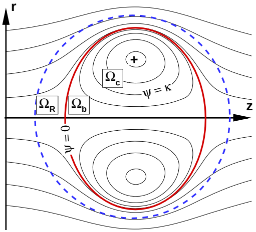
$$\begin{cases} \mathcal{L}\psi = & \omega_0 f(x, \psi, X), \quad \text{in } \Omega_R, \\ \psi = & \psi_{exp}, \quad \text{on } \partial\Omega_R. \end{cases}$$

- Find the "best" matching parameters X
  quasi-Newton Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm
- vorticity amplitude (vortex intensity) ω_0

$$\Gamma_{exp} = \int_{\partial\Omega} \frac{1}{r} \frac{\partial \psi_{exp}}{\partial \vec{n}} dS = \omega_0 \int_{\Omega} f(x, \psi, X) dr dz.$$

Original approach for the vortex ring reconstruction

I. Danaila and B. Protasz, submitted, 2014.



$$\underset{f \in H^1(\Omega_R)}{\mathrm{Min}} \; J(\psi) = \int_{\partial\Omega_R} \left| \frac{1}{r} \left(\frac{\partial \psi(\mathbf{f})}{\partial \vec{n}} - \frac{\partial \psi_{exp}}{\partial \vec{n}} \right) \right|^2 \mathrm{d}S$$

subject to

$$\begin{cases} \mathcal{L}\psi = & \omega_0 \mathbf{f}(\psi), \quad \text{in } \Omega_{\mathrm{R}}, \\ \dfrac{\omega}{r} & = \mathbf{f}(\psi). \end{cases}$$

- Optimal problem as in shape optimization

  weighted Sobolev gradient methods for the minimization

Numerical algorithm

- validated against Hill and Norbury vortices,
- used for DNS generated vortex rings.

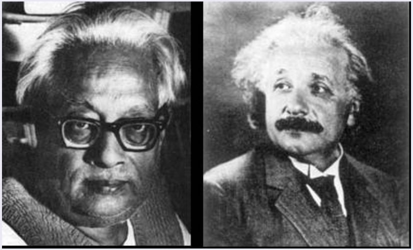
Outline

Bose-Einstein condensate

New state of the matter: super-atom
Properties: superfluid, super-conductor.

Predicted in 1924

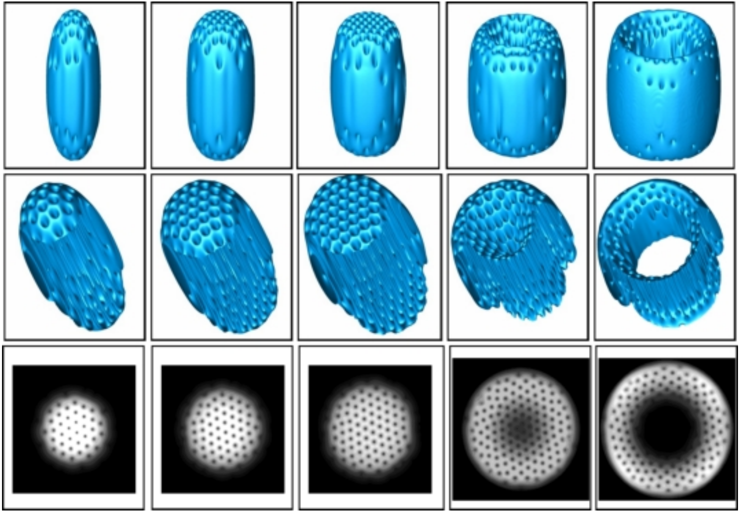
S. Bose A. Einstein



Created in 1995
Nobel Prize 2001

C. E. Wieman (Univ. Colorado)
E. A. Cornell (Univ. Colorado)
W. Ketterle (MIT, Cambridge)

Quantized vortices: 3D simulation of real experiments (I. Danaila, Phys. Rev. A, 2003, 2004, 2005.)



Outline

Conclusion

Thanks Paolo for the DNS method and code in cylindrical coordinates!
They proved useful for many physical and mathematical problems!

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```
int i=70;
while (i)
{        cout<<" Happy birthday !" << i << endl;
         i++;
}
```