IV

Modeling and Analysis of Heart Murmurs

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Cardiac Auscultation

"But..."

• Low specificity (high false positives)
• Diagnosis is based on the empirical/statistical correlation
• Source mechanism of murmurs is poorly understood
• No modality provides simultaneous assessment of source and measurement

60% of all pediatric murmurs leading to referral are “innocent”
Computational Hemo-acoustics

Can computational modeling provide the missing link between cause (pathology) and effect (sound)?

**Computational Hemo-acoustics (CHA) directly simulate the above procedure:**
- Prediction of murmur generation/propagation
- Source mechanism of murmurs
- Better Disease - Hemodynamics - Sound (Auscultation) relation

**Present Approach:**
- **Immersed Boundary Method based Hybrid Approach**
  - **Blood Flow** - *IBM Incompressible Navier-Stokes solver*
  - **Flow induced sound** - *Linearized Perturbed Compressible Equations (LPCE)*
  - **Sound Propagation in tissue** – *Linear wave equation*
Incompressible N-S Eqns.
\[ \nabla \cdot \vec{U} = 0, \quad \frac{D\vec{U}}{Dt} + \frac{1}{\rho_0} \nabla P = \nu_0 \nabla^2 \vec{U} \]

Structural wave Eqns.
\[ \begin{align*}
\frac{\partial p_{ij}}{\partial t} + \lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) &= S_p \\
\frac{\partial u_i}{\partial t} + \frac{1}{\rho} \frac{\partial p_{ij}}{\partial x_j} &= \frac{\eta}{\rho} \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + S_{u,i}
\end{align*} \]
Murmur Associated with Aortic Stenosis

Aortic Valve Stenosis

Aortic Stenosis Murmur

Simplified Hemodynamic Modeling

75% Aortic valve stenosis
Cardio-Thoracic Phantom Studies

Material: EcoFlex-10

\[ \rho = 1040 \, \text{kg/m}^3 \]
\[ E = 55.16 \, \text{kPa} \]
\[ K = 91.3 \, \text{Mpa} \quad (c_b = 297.3 \, \text{m/s}) \]
\[ G = 18.39 \, \text{kPa} \quad (c_s = 4.2 \, \text{m/s}) \]
\[ \mu = 14 \, \text{Pa s} \]

Thoracic phantom (silicone gel)

\[ U = 0.25 \, \text{m/s} \]
\[ D = 1.5875 \, \text{cm} \]
\[ D_T = 9.84 \, \text{cm} \]

Re = \( UD/\nu = 4000 \)
St = \( fD/U \)
Acoustic Sensors

Biopac sensor attached to the Micromanipulator

HP sensor attached to the Micromanipulator
Silicone Rubber- Tissue Mimicking Material

• Silicone rubber, Ecoflex 010 (Smooth-on)
  – Easy to produce
  – Extremely stable
  – Non-toxic and
  – Negligible shrinkage

• Procedure to make
  – Mixing Part A part B,
  – Adding Silicon thinner,
  – Degassing for 3-4 min in (-29 in Hg) to remove air bubbles
Murmur Generating

3D printed Casts

PVC Pipe

3D Printed Cast

Gripper

Stenosis

Flow fluctuation

Wave propagation

U

D
Biopac sensor attached to the Micromanipulator

HP sensor attached to the Micromanipulator
Cardiothoracic Phantom-2\textsuperscript{nd} generation

- Adding lung to the phantom
- Foam is used to model the lung
- Non-axisymmetric model
Experimental Measurements

Outer-surface radial accelerations

Frequency spectrum

Energy mapping
Simple model for the aortic stenosis murmur

Thoracic phantom (silicone gel)

Stenosis
Flow fluctuation
Wave propagation

Material properties:
Tissue mimicking, viscoelastic gel (EcoFlex-10)

\[ \rho = 1040 \text{ kg/m}^3 \]
\[ K = 1.04 \text{ GPa} \quad (c_b = 1000.0 \text{ m/s}) \]
\[ G = 18.39 \text{ kPa} \quad (c_s = 4.2 \text{ m/s}) \]
\[ \mu = 14 \text{ Pa s} \]

Other parameters:
\[ U = 0.25 \text{ m/s} \]
\[ D = 1.5875 \text{ cm} \]
\[ D_T = 9.84 \text{ cm (gelA), 16.51 cm (gelB)} \]

\( c.f. \)

Biological soft tissue:
\[ K = 2.25 \text{ GPa} \quad (c_b = 1500 \text{ m/s}) \]
\[ G = 0.1 \text{ MPa} \quad (c_s = 10 \text{ m/s}) \]
\[ \mu = 0.5 \text{ Pa s} \]

Re = \( UD / \nu = 4000 \)
Computational Modeling

Elastic wave eq.
for viscoelastic material

Generalized Hooke’s law
Kelvin-Voigt model

\[
\frac{\partial p'_i}{\partial t} + \lambda \frac{\partial u'_i}{\partial x_j} \delta_{ij} + \mu \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) = 0
\]

\[
\frac{\partial u'_i}{\partial t} + \frac{1}{\rho} \frac{\partial p'_i}{\partial x_j} = \eta \frac{\partial}{\partial x_j} \left( \frac{\partial u'_i}{\partial x_i} + \frac{\partial u'_j}{\partial x_i} \right)
\]

High-order IBM,
6th order Compact Finite
Difference Scheme,
4 stage Runge-Kutta method

Hemodynamics
IBM, Incompressible N-S

\[
\nabla \cdot \vec{U} = 0, \quad \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla)\vec{U} + \frac{1}{\rho} \nabla P = \nu \nabla^2 \vec{U}
\]
Flow Simulation

$Re_D = 4000$

**Axial velocity**

**Vorticity**

**Pressure**

Wall pressure spectrum

- $-1.0D$
- $-0.5D$
- $0$
- $0.5D$
- $1.0D$
- $1.5D$
- $2.0D$
- $2.5D$
- $3.0D$
- $3.5D$
- $4.0D$

$FREQ \ [Hz]$
3D Elastic Wave Simulation

Radial velocity fluctuation contours

- 200x200x320 (12.8 M), about 60 hrs with 1024 cores for real time
  0.8 sec
Comparison with Experimental Measurements

Outer-surface radial accelerations

Frequency spectrum

Energy mapping
Analytical estimation of elastic wave solution (no geometrical effects)

\[ \rho \frac{\partial^2 u_i}{\partial t^2} - \left( \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \right) \frac{\partial^2 u_i}{\partial x_j \partial x_k} = f_i(t) \delta(\vec{r}) \]

\[ u_i(t) = \frac{1}{2\pi} \int U_i(\omega) e^{-i\omega t} d\omega \]

\[ f_i(t) = \frac{1}{2\pi} \int F_i(\omega) e^{-i\omega t} d\omega \]

\[ U_i(\vec{r}, \omega) = G_y(\vec{r}, \omega) F_j(\omega) \]

Green’s tensor (Ben-Menahem & Singh, 1981)

\[ G_{ij}(\vec{r}, \omega) = \frac{ik_p}{12\pi(\lambda + 2\mu)} \left( \delta_{ij} h_0^{(1)}(k_p r) + (\delta_{ij} - \frac{3x_i x_j}{r^2}) h_2^{(1)}(k_p r) \right) \]

\[ -\frac{ik_s}{12\pi \mu} \left( -2\delta_{ij} h_0^{(1)}(k_s r) + (\delta_{ij} - \frac{3x_i x_j}{r^2}) h_2^{(1)}(k_s r) \right) \]

\[ k_p = \omega / c_p, \quad c_p = \sqrt{(\lambda + 2\mu) / \rho} \]

\[ k_s = \omega / c_s, \quad c_s = \sqrt{\mu / \rho} \]
Evaluation of Radial Acceleration

\[ a_m(\vec{r}, \omega) = \sum_k (i\omega)^2 G_{mn,k}(\vec{r}_k, \omega) F_{n,k}(\omega), \quad F_{2,k}(\omega) = P_k(\omega) \Delta A \]

\[ a_2(\omega) \]

Oblique shear waves contribute significantly to the stethoscopic signal!
Source Localization

Surface measurements

\[ u_1, u_2, \ldots, u_M \]

Source mapping

\[ F_1, F_2, \ldots, F_N \]

\[ u_m = \sum_{n=1}^{N} G(x_m; x_n)F_n \]

\[
\begin{bmatrix}
    u_1 \\
    \vdots \\
    u_M
\end{bmatrix} =
G
\begin{bmatrix}
    F_1 \\
    \vdots \\
    F_N
\end{bmatrix}
\]

\( G \): M by N complex matrix

\( G^+ \): Pseudo inverse of G
Proceeding towards using a multi-sensor stethoscopic array (StethoVest) for automatic murmur localization.
Computational Modeling

Aortic stenosis

Patient specific geometry Model

\[ U = 0.25 \text{ m/s} \]

\[ \text{Re} = \frac{UD}{\nu} = 4000 \]

\[ D = 1.5875 \text{ cm} \]

Epidermal surface
Fluid Solver:

Vicar Code, sharp interface IBM

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = - \frac{1}{\rho_f} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2}
\]

\[
\frac{\partial U_i}{\partial x_i} = 0.
\]

See: Mittal, R., et al., JCP, 2008

Acoustic Solver:

\[
\frac{\partial p_{ij}}{\partial t} + \lambda \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) = 0
\]

\[
\frac{\partial v_i}{\partial t} + \frac{1}{\rho_s} \frac{\partial p_{ij}}{\partial x_j} = \frac{\eta}{\rho_s} \frac{\partial}{\partial x_j} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).
\]

\(v_i\) – structure velocity.
\(\lambda, \mu\) – 1\textsuperscript{st} and 2\textsuperscript{nd} Lame’s constants.
\(\rho_s\) – density.
\(K\) – bulk modulus, = 1.04 GPa.
\(G\) – shear modulus, = 18.39 KPa.
\(\eta\) – viscosity, = 14.0 Pa s.

Numerical methods:

Interior nodes:
6\textsuperscript{th} – order compact scheme

Immersed boundary:
approximating polynomial method

Time advancement:
4\textsuperscript{th} – order Runge-Kutta method

See: Seo, J. H., & Mittal, R., JCP, 2011
Hemodynamic Simulation Results

x component of vorticity

50%

75%

90%

Same contour level
Hemodynamic Simulation Results

Contour of surface pressure

50%  75%  90%

5 times smaller contour level  Baseline  5 times larger contour level
Source Location

Surface Signal

50%

1000 times smaller contour level

75%

Baseline

90%

100 times larger contour level
Realistic Thorax Model

Need to account for thoracic structures on sound propagation

CT scan data (Visible Human) → Material property (density and speed of sound) mapping → 3D model of the thorax (density iso-surfaces)
Cited Paper - Hemoacoustics


